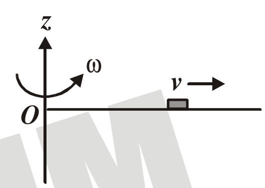
IIT-JEE 2012 Solution of Paper -1

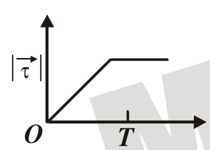
PART I : PHYSICS

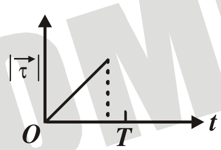
SECTION I : Single Correct Answer Type

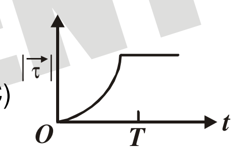
This section contains 10 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

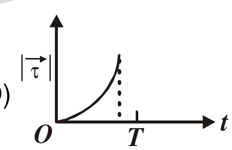
1. A thin uniform rod, pivoted at O, is rotating in the horizontal plane with constant angular speed ω, as shown in the figure. At time $t = 0$, a small insect starts from O and moves with constant speed v with respect to the rod towards the other end. It reaches the end of the rod at $t = T$ and stops. The angular speed of the system remains ω throughout. The magnitude of the torque $(|\vec{\tau}|)$ on the system about O, as a function of time is best represented by which plot ?



(A) 

(B) 

(C) 

(D) 

1.(B) $L = \left(I_{rod} + mx^2\right)\omega$

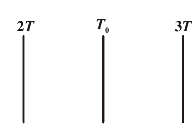
$$\tau = \frac{dL}{dt} = 2m\omega xv = 2m\omega v^2 t$$

2. Three very large plates of same area are kept parallel and close to each other. They are considered as ideal black surfaces and have very high thermal conductivity. The first and third plates are maintained at temperatures $2T$ and $3T$ respectively. The temperature of the middle (i.e. second) plate under steady state condition is:

(A) $\left(\frac{65}{2}\right)^{\frac{1}{4}} T$ (B) $\left(\frac{97}{4}\right)^{\frac{1}{4}} T$ (C) $\left(\frac{97}{2}\right)^{\frac{1}{4}} T$ (D) $(97)^{\frac{1}{4}} T$

2.(C) $(3T)^4 - T_0^4 = T_0^4 - (2T)^4$

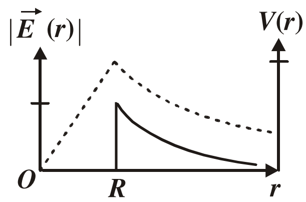
$\Rightarrow 2T_0^4 = 97T^4$



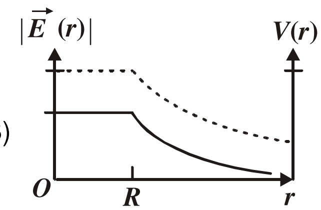
$$T_0 = \left(\frac{97}{2}\right)^{1/4} T$$

3. Consider a thin spherical shell of radius R with its centre at the origin, carrying uniform positive surface charge density. The variation of the magnitude of the electric field $|\vec{E}(r)|$ and the electric potential $V(r)$ with the distance r from the centre, is best represented by which graph?

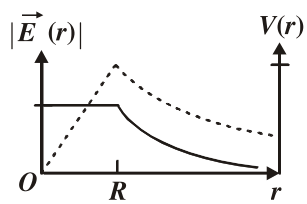
(A)



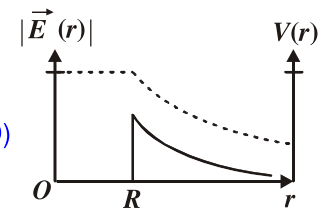
(B)



(C)



(D)



3.(D)

4. In the determination of Young's modulus $\left(Y = \frac{4MLg}{\pi l d^2}\right)$ by using Searle's method, a wire of length $L = 2m$ and diameter $d = 0.5mm$ is used. For a load $M = 2.5kg$, an extension $l = 0.25mm$ in the length of the wire is observed. Quantities d and l are measured using a screw gauge and a micrometer, respectively. They have the same pitch of $0.5mm$. The number of divisions on their circular scale is 100. The contributions to the maximum probable error of the Y measurement :

(A) due to the errors in the measurements of d and l are the same

(B) due to the error in the measurement of d is twice that due to the error in the measurement of l

(C) due to the error in the measurement of l is twice that due to the error in the measurement of d

(D) due to the error in the measurement of d is four times that due to the error in the measurement of l

4.(A) $Y = \frac{4MLg}{\pi l d^2}$

$$\frac{\Delta Y}{Y} = \frac{\Delta M}{M} + \frac{\Delta L}{L} + \frac{\Delta l}{l} + 2\frac{\Delta d}{d}$$

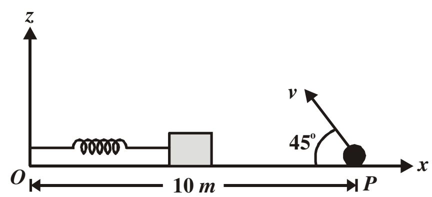
$d = 0.5mm$, $l = 0.25mm$

L.C for d = L.C. for l

$\therefore \Delta l = \Delta d \qquad \therefore \frac{\Delta l}{l} = \frac{\Delta l}{0.25}$

$$2 \cdot \frac{\Delta d}{d} = \frac{2 \cdot \Delta l}{0.5} = \frac{\Delta l}{0.25}$$

5. A small block is connected to one end of a massless spring of un-stretched length $4.9\,m$. The other end of the spring (see the figure) is fixed. The system lies on a horizontal frictionless surface. The block is stretched by $0.2\,m$ and released from rest at $t = 0$. It then executes simple harmonic motion with angular frequency $\omega = \frac{\pi}{3}\,\text{rad/sec}$. Simultaneously at $t = 0$, a small pebble is projected with speed v from point P at an angle of 45^o as shown in the figure. Point P is at a horizontal distance of $10\,m$ from O. If the pebble hits the block at $t = 1\,s$, the value of v is (take $g = 10\,m/s^2$)



(A) $\sqrt{50}\,m/s$ (B) $\sqrt{51}\,m/s$ (C) $\sqrt{52}\,m/s$ (D) $\sqrt{53}\,m/s$

5.(A) Time of flight $= 1\,\text{sec}$

$$\frac{2 \cdot v}{\sqrt{2}\,g} = 1 \qquad \Rightarrow v = \frac{g}{\sqrt{2}} = \sqrt{50}\,m/s$$

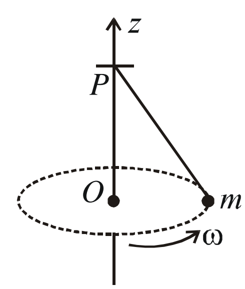
6. Young's double slit experiment is carried out by using green, red and blue light, one color at a time. The fringe widths recorded are β_G, β_R and β_B, respectively. Then,

(A) $\beta_G > \beta_B > B_R$ (B) $\beta_B > \beta_G > B_R$

(C) $\beta_R > \beta_B > B_G$ (D) $\beta_R > \beta_G > B_B$

6.(D) $\lambda_R > \lambda_G > \lambda_B \qquad \therefore \beta_R > \beta_G > \beta_B$

7. A small mass m is attached to a massless string whose other end is fixed at P as shown in the figure. The mass is under going circular motion in the $x - y$ plane with centre at O and constant angular speed ω. If the angular momentum of the system, calculated about O and P are denoted by $\vec{L}_0$ and $\vec{L}_P$ respectively, then



(A) $\vec{L}_O$ and $\vec{L}_P$ do not vary with time.

(B) $\vec{L}_O$ varies with time while $\vec{L}_P$ remains constant.

(C) $\vec{L}_O$ remains constant while $\vec{L}_P$ varies with time.

(D) $\vec{L}_O$ and $\vec{L}_P$ both vary with time.

7.(C) $\vec{L} = \vec{r} \times \vec{p}$

L_0 is always along z-axis

L_p rotates about z-axis

8. A mixture of 2 moles of helium gas (atomic mass = 4 amu) and 1 mole of argon gas (atomic mass = 40 amu) is kept at 300 K in a container. The ratio of the r m s speeds $\left(\frac{v_{rms}(helium)}{v_{rms}(\arg on)}\right)$ is

(A) 0.32 (B) 0.45 (C) 2.24 (D) 3.16

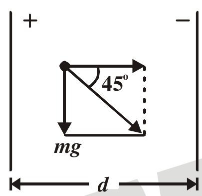
8.(D) $V_{rms} = \sqrt{\frac{\gamma RT}{M_0}}$

$\frac{V_{He}}{V_{Ar}} = \sqrt{\frac{40}{4}} = \sqrt{10}$

9. Two large vertical and paralle metal plates having a separation of 1 cm are connected to a DC voltage source of potential difference X. A proton is released at rest midway between the two plates. It is found to move at 45^0 to the vertical JUST after release. Then X is nearly

(A) $1\times10^{-5}V$ (B) $1\times10^{-7}V$ (C) $1\times10^{-9}V$ (D) $1\times10^{-10}V$

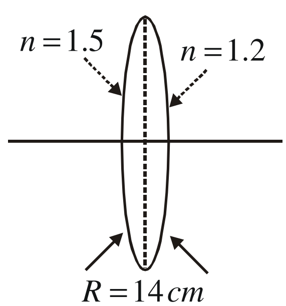
9.(C) $\frac{eE}{mg} = \tan 45^o$



$E = \frac{mg}{e}$

$V = Ed = \frac{mgd}{e} = 10^{-9}V$

10. A bi-convex lens is formed with two thin plano-convex lenses as shown in the figure. Refractive index n of the first lens is 1.5 and that of the second lens is 1.2. Both the curved surfaces are of the same radius of curvature $R = 14\,cm.$ For this bi-convex lens, for an object distance of 40 cm, the image distance will be



(A) $-280.0\,cm$ (B) $40.0\,cm$ (C) $21.5\,cm$ (D) $13.3\,cm$

10.(B) Focal length of plane-convex lens

$\frac{1}{f_1} = (1.5-1)\left(\frac{1}{14}\right) = \frac{1}{28}$

$f_1 = 28\,cm$

$$\frac{1}{f_2} = (1.2-1)\left(\frac{1}{14}\right) = \frac{1}{70}$$

$$f_2 = 70\,cm$$

Focal length of combination

$$\frac{1}{f} = \frac{1}{28} + \frac{1}{70} = \frac{7}{140} = \frac{1}{20}$$

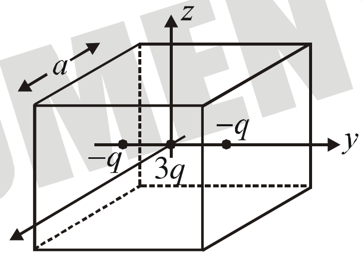
$$\therefore \frac{1}{V} + \frac{1}{40} = \frac{1}{20}$$

$$\frac{1}{V} = \frac{1}{40}, \qquad V = 40\,cm$$

SECTION II : Multiple Correct Answer(s) Type

This section contains 5 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

11. A cubical region of side a has its centre at the origin. It encloses three fixed point charges, $-q$ at $(0,-a/4,0), +3q$ at (0,0,0) and $-q$ at $(0,+a/4,0)$. Choose the correct option(s)



(A) The net electric flux crossing the plane $x=+a/2$ is equal to the net electric flux crossing the plane $x=-a/2$

(B) The net electric flux crossing the plane $y=+a/2$ is more than the net electric flux crossing the plane $y=-a/2$

(C) The net electric flux crossing the entire region is $\frac{q}{\varepsilon_0}$.

(D) The net electric flux crossing the plane $z=+a/2$ is equal to the net electric flux crossing the plane $x=+a/2$

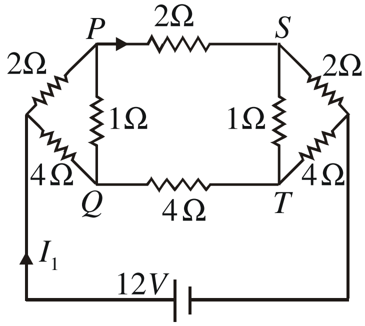
11.(A,C,D)

$A \rightarrow$ because both planes are symmetrically located with respect to charge $B \rightarrow$ both planes are once again symmetrical

$C \rightarrow$ Gauss law

$D \rightarrow$ because both are symmetrical

12. For the resistance network shown in the figure, choose the correct option(s)



(A) The current through PQ is zero

(B) $I_1 = 3A$

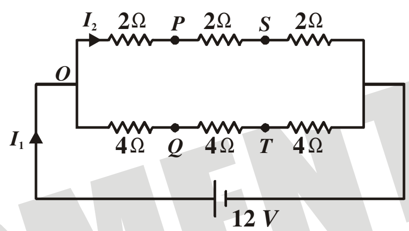
(C) The potential at S is less than that at Q

(D) $I_2 = 2A$

12.(A,B,C,D)

From symmetry current through PQ & ST is zero.

$\therefore$ circuit is

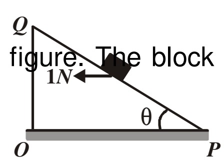


$R_{eq} = 4\Omega$ $\quad \therefore I_1 = 3A$

$$I_2 = \frac{12}{18} \times 3 = 2A$$

$V_S = V_0 - 4I_2 = V_0 - 8,$ $\quad V_Q = V_0 - 1 \times 4 = V_0 - 4$

13. A small block of mass of 0.1 Kg lies on a fixed inclined plane PQ which makes an angle θ with the horizontal. A horizontal force of $1N$ acts on the block through its center of mass as shown in the figure. The block remains stationary if



(take $g = 10 m/s^2$)

(A) $\theta = 45^0$

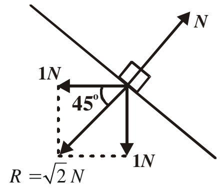
(B) $\theta > 45^0$ and a frictional force acts on the block towards P.

(C) $\theta > 45^0$ and a frictional force acts on the block towards Q.

(D) $\theta < 45^0$ and a frictional force acts on the block towards Q.

13.(A,C)

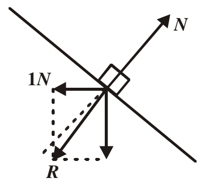
For $\theta = 45^o$



There is no net force along the incline

$\therefore$ friction will be zero & block at rest

If $\theta > 45^o$ then resultant of $1N$ and $1N$ will have a component down the incline.



14. Consider the motion of a positive point charge in aregion where there are simultaneous uniform electric and magnetic fields $\vec{E} = E_0\hat{j}$ and $\vec{B} = B_0\hat{j}$. At time $t = 0$, this charge has velocity $\vec{v}$ in the $x - y$ plane, making an angle θ with the x-axis. Which of the following option(s) is (are) correct for time $t > 0$?

(A) If $\theta = 0^o$, the charge moves in a circular path in the $x - z$ plane

(B) If $\theta = 0^o$, the charge undergoes helical motion with constant pitch along the y-axis

(C) If $\theta = 10^o$, the charge undergoes helical motion with its pitch increasing with time, along the y-axis

(D) If $\theta = 90^o$, the charge undergoes linear but accelerated motion along the y-axis

14.(C,D)

15. A person blows into open-end of a long pipe. As a result, a high-pressure pulse of air travels down the pipe. When this pulse reaches the other end of the pipe :

(A) a high-pressure pulse starts traveling up the pipe, if the other end of the pipe is open

(B) a low-pressure pulse starts traveling up the pipe, if the other end of the pipe is open

(C) a low-pressure pulse starts traveling up the pipe, if the other end of the pipe is closed (D) a high-pressure pulse starts traveling up the pipe, if the other end of the pipe is closed

15.(B,D)

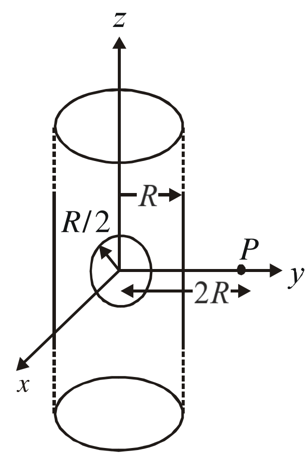
At open end reflected pressure wave suffers a phase change of π.

At closed end there is no phase change.

SECTION III : Integer Answer Type

This section contains 5 questions. The answer to each question is a single digit integer, ranging from 0 to 9 (both inclusive).

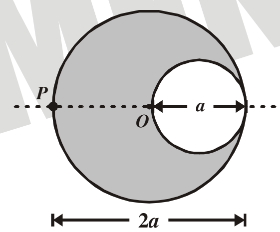
16. An infinitely long solid cylinder of radius R has a uniform volume charge density ρ. It has a spherical cavity of radius $R/2$ with its centre on the axis of the cylinder, as shown in the figure. The magnitude of the electric field at the point P, which is at a distance $2R$ from the axis of the cylinder, is given by the expression $\frac{23\rho R}{16k\varepsilon_0}$. The value of k is :



16.(6) $E_P = E_{Cylinder} - E_{Sphere}$

$$= \frac{\lambda}{2\pi\varepsilon_0(2R)} - \frac{Q}{4\pi\varepsilon_0(2R)^2} \qquad = \frac{\rho \cdot R^2}{4\varepsilon_0 R} - \frac{\frac{4}{3}\pi\left(\frac{R}{2}\right)^3 \rho}{16\pi\varepsilon_0 R^2} \qquad = \frac{\rho R}{4\varepsilon_0} - \frac{R\rho}{96\varepsilon_0} = \frac{23\rho R}{96\varepsilon_0}$$

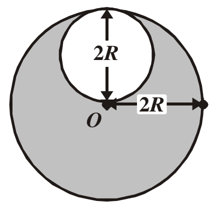
17. A cylindrical cavity of diameter a exists inside a cylinder of diameter $2a$ as shown in the figure. Both the cylinder and the cavity are infinitely long. A uniform current density J flows along the length. If the magnitude of the magnetic field at the point P is given by $\frac{N}{12}\mu_0 aJ$, then the value of N is :



17.(5) $B_P = B_{Cylinder} - B_{Cavity}$

$$= \frac{\mu_0\left[J \cdot \pi a^2\right]}{2\pi a} - \frac{\mu_0\left[J \cdot \pi \frac{a^2}{4}\right]}{2 \cdot \frac{3a}{2}\pi} \qquad = \mu_0 aJ\left[\frac{1}{2} - \frac{1}{12}\right] = \frac{5}{12}\mu_0 aJ$$

18. A lamina is made by removing a small disc of diameter $2R$ from a bigger disc of uniform mass density and radius $2R$, as shown in the figure. The moment of inertia of this lamina about axes passing through O and P is I_0 and I_p, respectively. Both these axes are perpendicular to the plane of the lamina. The ratio I_p / I_O to the nearest integer is :

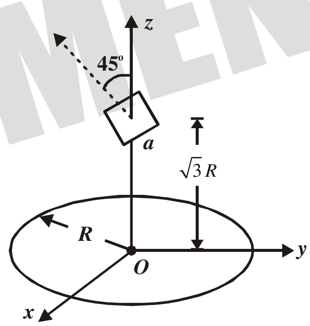


18.(3) $I_0 = \frac{4M\left(2R\right)^2}{2} - \left[\frac{MR^2}{2} + MR^2\right] \quad = \frac{13}{2}MR^2$ [$M =$ mass of cavity]

$I_p = \frac{3}{2}(4M)(2R)^2 - \left[\frac{MR^2}{2} + M\left(5R^2\right)\right] \quad = \frac{37}{2}\pi R^2$

$\therefore \frac{I_p}{I_0} = \frac{37}{13} \approx 3$

19. A circular wire loop of radius R is placed in the $x-y$ plane centered at the origin O. A square loop of side $a\,(a << R)$ having two turns is placed with its center at $z = \sqrt{3}\,R$ along the axis of the circular wire loop, as shown in figure. The plane of the square loop makes an angle of 45^o with respect to the z-axis. If the mutual inductance between the loops is given by $\frac{\mu_0 a^2}{2^{p/2}R}$, then the value of p is :



19.(7) $B = \frac{\mu_0 I R^2}{2\left(R^2 + 3R^2\right)^{3/2}}$

[$I =$ current in circular loop] $\quad = \frac{\mu_0 I}{16R}$

Flux through square loop

$\phi = NAB\cos 45^o \quad = 2a^2\frac{\mu_0 I}{16R}\cdot\frac{1}{\sqrt{2}} = \frac{\mu_0 a^2}{2^{7/2}R}\cdot I$

$\therefore M = \frac{\mu_0 a^2}{2^{7/2}R}$

20. A proton is fired from very far away towards a nucleus with charge $Q = 120e$, where e is the electronic charge. It makes a closest approach of $10\ fm$ to the nucleus. The de-Broglie wavelength (in units of fm) of the proton at its start is : (take the proton mass, $m_p = (5/3)\times 10^{-27}\ kg$,

$h/e = 4.2\times 10^{-15}\ J\cdot s/C$,

$1/4\pi\varepsilon_0 = 9\times 10^9\ N.m^2/C^2$,

$1\ fm = 10^{-15}\ m$)

20.(7) $K = KE$ at start

$$K = \frac{1}{4\pi\varepsilon_0}\frac{120e^2}{\left(10^{-14}\right)} = \frac{9\times 10^9\times 120\times e^2}{10^{-14}} \qquad = 108\times 10^{24}\times e^2$$

$$P = \sqrt{2mK}$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2\times\frac{5}{3}\times 10^{-27}\times 108\times 10^{24}\times e^2}} \qquad = \frac{4.2\times 10^{-15}}{6\times 10^{-1}} = 7\times 10^{-15} = 7\ fm$$

PART II : CHEMISTRY

SECTION I : Single Correct Answer Type

This section contains 10 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

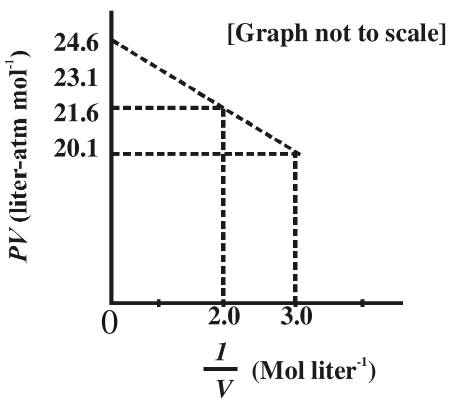
21. In allene (C_3H_4), the type(s) of hybridisation of the carbon atoms is (are)

(A) sp and sp^3 (B) sp and sp^2 (C) Only sp^2 (D) sp^2 and sp^3

21.(B)

$$\overset{sp^2}{H-\underset{\displaystyle H}{\overset{|}{C}}} = \overset{sp}{C} = \underset{\displaystyle H}{\overset{sp^2}{C}}-H$$

22. For one mole of a van der Waals gas when $b = 0$ and $T = 300\ K$, the PV vs $1/V$ plot is shown below. The value of the van der Waals constant a $(atm.liter^2\ mol^{-2})$ is



(A) 1.0 (B) 4.5 (C) 1.5 (D) 3.0

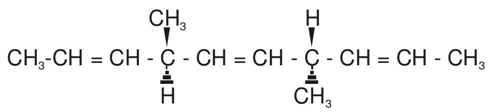
22.(C) $\left(P+\frac{a}{V^2}\right)(V)=RT$

$$PV+\frac{a}{V}=RT$$

$$PV=RT-\frac{a}{V}$$

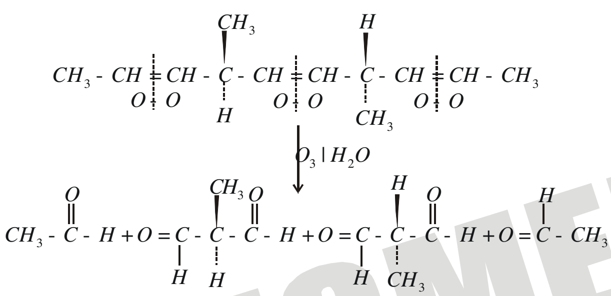
Slope = $-a$ $\quad =\frac{y_2-y_1}{x_2-x_1} \quad =\frac{21.6-20.1}{3-2} \quad =1.5$

23. The number of optically active products obtained from the **complete** ozonolysis of the given compound is



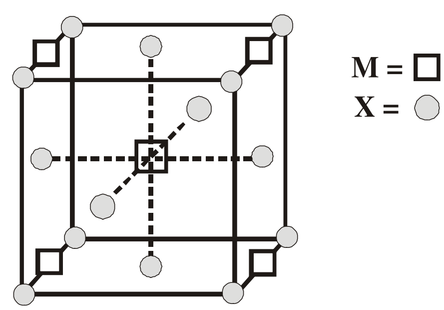
(A) 0 (B) 1 (C) 2 (D) 4

23.(A)



(A) is correct i.e. none is optically acitive

24. A compound M_pX_q has cubic close packing (ccp) arrangement of X. Its unit cell structure is shown below. The empirical formula of the compound is



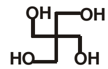
(A) MX (B) MX_2 (C) M_2X (D) M_5X_{14}

24.(B) $M=4\times\frac{1}{4}+1=2$

$$X=8\times\frac{1}{8}+6\times\frac{1}{2}=4$$

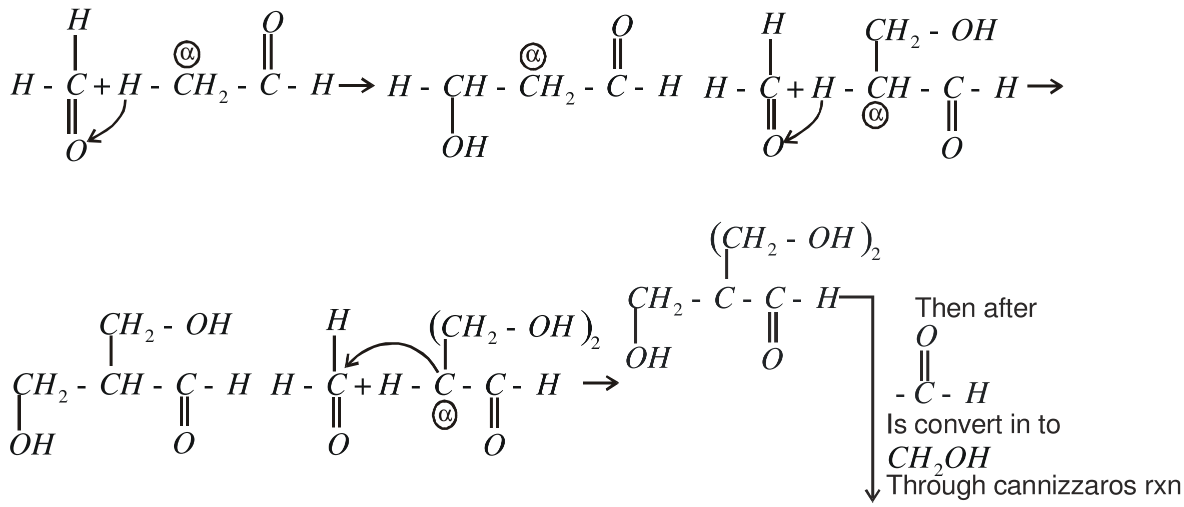
$$MX_2$$

25. The number of aldol reaction (s) that occurs in the given transformation is

$CH_3CHO + 4HCHO \xrightarrow{conc.aq.NaOH}$ 

(A) 1 (B) 2 (C) 3 (D) 4

25.(C)



$\therefore$ (C) is correct

26. The colour of light absorbed by an aqueous solution of $CuSO_4$ is

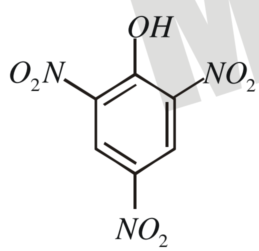
(A) orange-red (B) blue-green (C) yellow (D) violet

26.(A) Orange Red

27. The carboxyl functional group $(-COOH)$ is present in

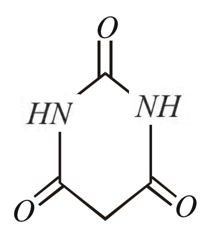
(A) picric acid (B) barbituric acid (C) ascorbic acid (D) aspirin

27.(D)

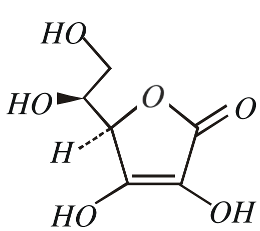


Picric acid

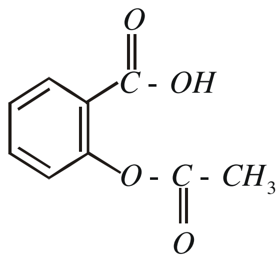
(2,4,6- Trinitrophenol)



Barbituric acid



Ascorbic acid



Aspirin (O-Acetyl salicylic acid)

28. The kinetic energy of an electron in the second Bohr orbit of a hydrogen atom is [a_0 is Bohr radius]

(A) $\dfrac{h^2}{4\pi^2 m a_0^2}$ (B) $\dfrac{h^2}{16\pi^2 m a_0^2}$ (C) $\dfrac{h^2}{32\pi^2 m a_0^2}$ (D) $\dfrac{h^2}{64\pi^2 m a_0^2}$

28.(C) $m\nu r = 2\dfrac{h}{2\pi}$

$\dfrac{1}{2}m\nu^2 = \dfrac{1}{2}\dfrac{h^2}{m\pi^2 r^2}$

$r = 4a_0$ $\qquad K.E. = \dfrac{1}{2}\times\dfrac{h^2}{16m\pi^2 {a_0}^2}$

29. Which ordering of compounds is according to the decreasing order of the oxidation state of nitrogen?

(A) $HNO_3, NO, NH_4\ CI, N_2$ (B) $HNO_3, NO, N_2, NH_4\ CI$

(C) HNO_3, NH_4CI, NO, N_2 (D) NO, HNO_3, NH_4CI, N_2

29.(B) $HNO_3; NO; N_2; NH_4Cl$
+5 +2 0 -3

30. As per $IUPAC$ nomenclature, the name of the complex $\left[Co(H_2O)_4\ (NH_3)_2\right]\ CI_3$ is

(A) Tetraaquadiaminecobalt (III) chloride (B) Tetraaquadiamminecobalt (III) chloride
(C) Diaminetetraaquacobalt (III) chloride (D) Diamminetetraaquacobalt (III) chloride

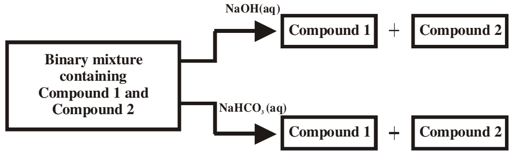
30.(D) $\left[Co\left(H_2O\right)_4\left(NH_3\right)_2\right]Cl_3$

Diamminetetraaquacobalt (III) chloride

SECTION II : Multiple Correct Answer(s) Type

This section contains 5 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

31. Identify the binary mixture (s) that can be separated into individual compounds, by differential extraction, as shown in the given scheme.



(A) C_6H_5OH and C_6H_5COOH

(B) C_6H_5COOH and $C_6H_5CH_2OH$

(C) $C_6H_5CH_2OH$ and C_6H_5OH

(D) $C_6H_5CH_2OH$ and $C_6H_5CH_2COOH$

31.(B,D)

In option (A)

Both C_6H_5OH & C_6H_5COOH both can react with $NaOH$ & $NaHCO_3$

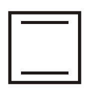
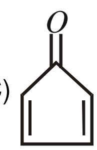
hence cannot be seperated.

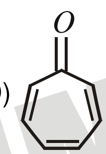
32. Choose the correct reason (s) for the stability of the **lyophobic** colloidal particles.

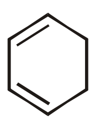
(A) Preferential adsorption of ions on their surface from the solution
(B) Preferential adsorption of solvent on their surface from the solution
(C) Attraction between different particles having opposite charges on their surface
(D) Potential difference between the fixed layer and the diffused layer of opposite charges around the colloidal particles

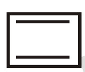
32. (A,C,D)

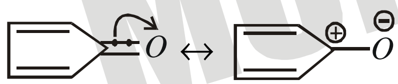
33. Which of the following molecules, in pure form, is (are) **unstable** at room temperature?

(A)  (B)  (C) 

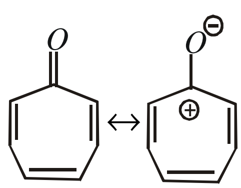
(D) 

33. (B,C)  stable ∵ of conjugated system

(B)  Unstable ∵ of conjugated system but antiaromatic

(C) 

Unstable ∵ $4\pi e^-$ i.e. antiaromatic polarisation

(D) 

stable ∵ $6\pi e^-$ aromatic conjugated system

∴ (b,c) is correct

34. Which of the following hydrogen halides react(s) with $AgNO_3$ (aq) to give a precipitate that dissolves in $Na_2S_2O_3(aq)$?

(A) HCl (B) HF (C) HBr (D) HI

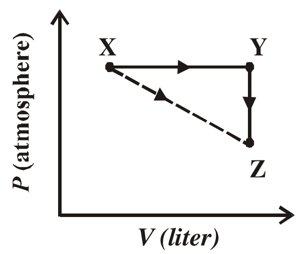
34.(A,C,D)

$$AgBr + 2Na_2S_2O_3 \rightarrow Na_3\left[Ag\left(S_2O_3\right)_2\right] + NaBr$$

$$AgCl + 2Na_2S_2O_3 \rightarrow Na_3\left[Ag\left(S_2O_3\right)_2\right] + NaCl$$

$$AgI + 2Na_2S_2O_3 \rightarrow Na_3\left[Ag\left(S_2O_3\right)_2\right] + NaI$$

35. For an ideal gas, consider only $P-V$ work in going from an initial state ***X*** to the final state Z. The final state Z can be reached by either of the two paths shown in the figure. Which of the following choice (s) is (are) correct ? [take ΔS as change in entropy and w as work done]



(A) $\Delta S_{x\rightarrow z} = \Delta S_{x\rightarrow y} + \Delta S_{y\rightarrow z}$ (B) $w_{x\rightarrow z} = w_{x\rightarrow y} + w_{y\rightarrow z}$

(C) $w_{x\rightarrow y\rightarrow z} = w_{x\rightarrow y}$ (D) $\Delta S_{x\rightarrow y\rightarrow z} = \Delta S_{x\rightarrow y}$

35.(AC) (A) Entropy is state function.
(C) Work done is area under the P-V curve.

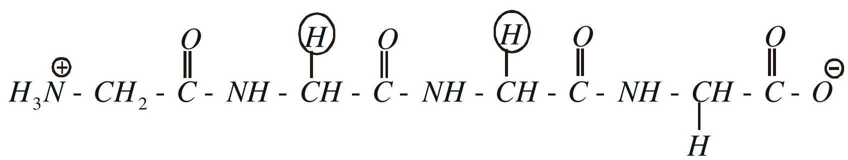
SECTION III : Integer Answer Type

This section contains 5 questions. The answer to each question is a single digit integer, ranging from 0 to 9 (both inclusive).

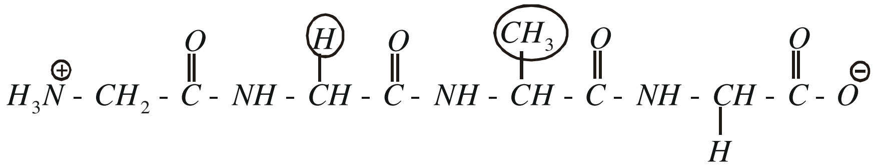
36. The substituents R_1 and R_2 for nine peptides are listed in the table given below. How many of these peptides are positively charged at pH = 7.0 ?

Peptide	R_1	R_2
I	H	H
II	H	CH_3
III	CH_2COOH	H
IV	CH_2CONH_2	$(CH_2)_4NH_2$
V	CH_2CONH_2	CH_2CONH_2
VI	$(CH_2)_4NH_2$	$(CH_2)_4NH_2$
VII	CH_2COOH	CH_2CONH_2
VIII	CH_2OH	$(CH_2)_4NH_2$
IX	$(CH_2)_4NH_2$	CH_3

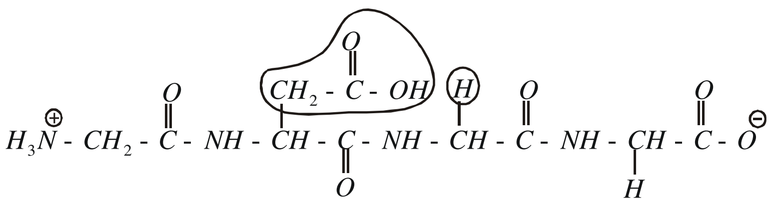
36.(4) **Peptide (I)**



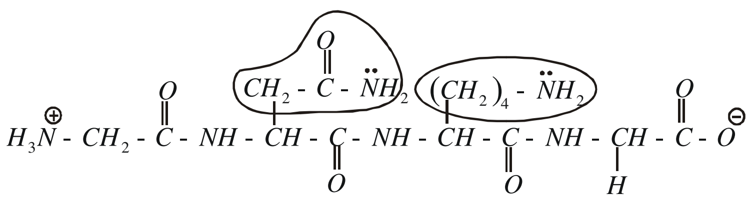
Peptide (II)



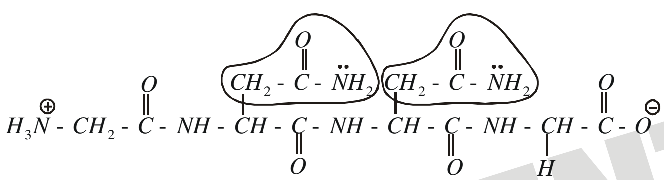
Peptide (III)



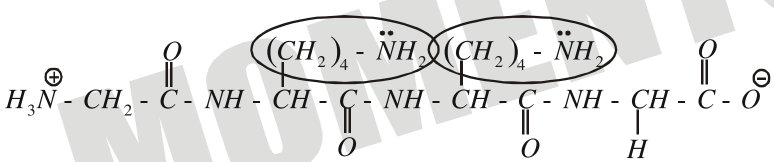
Peptide (IV)



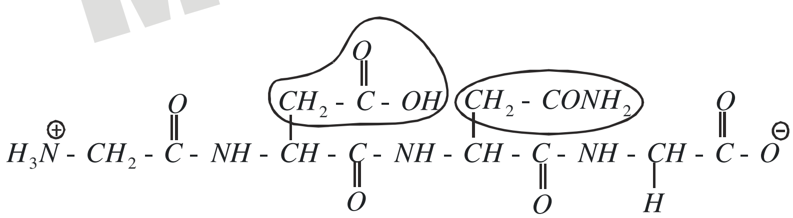
Peptide (V)



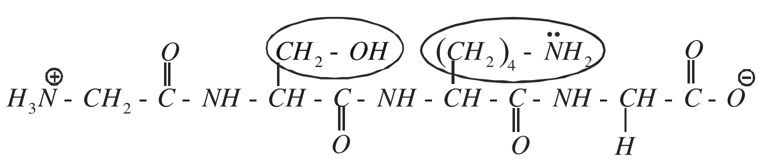
Peptide (VI)



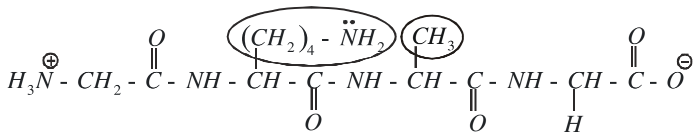
Peptide (VII)



Peptide (VIII)



Peptide (IX)



Ans . 3 Peptide IV /Peptide VI / Peptide VIII/Peptide IX because of lone pair of electron present on nitrogen which at pH = 7 undergoes protonation i.e. positive in nature.

37. The periodic table consists of 18 groups. An isotope of copper, on bombardment with protons, undergoes a nuclear reaction yielding element X as shown below. To which group, element X belongs in the periodic table?

$${}^{63}_{29}Cu + {}^{1}_{1}H \rightarrow 6\,{}^{1}_{0}n + \alpha + 2{}^{1}_{1}H + X$$

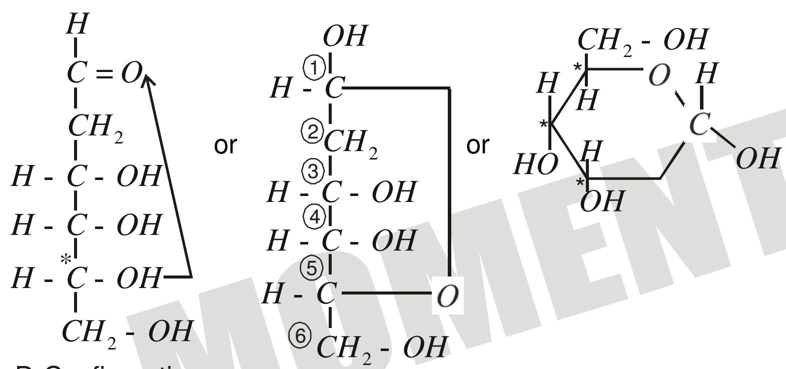
37.(8) $_{29}Cu^{63} +_1 H^1 \rightarrow 6_0 n^1 +_2 He^4 + 2_1 H^1 +_{26} Fe^{52}$

Fe in 8th group

38. When the following aldohexose exists in its D-configuration, the total number of stereoisomers in its pyranose form is

CHO
|
CH_2
|
CHOH
|
CHOH
|
CHOH
|
CH_2OH

38.(8)



D-Configuration

In Pyranose form total 3 chiral or stereocentre hence total $2^3 = 8$ stereo isomers.

39. 29.2% (w/w) HCI stock solution has a density of 1.25 $g\ mL^{-1}$. The molecular weight of HCI is 36.5 $g\ mol^{-1}$. The volume (mL) of stock solution required to prepare a 200 mL solution of $0.4\ M\ \ HCI$ is

39.(8) 29.2 % (w/w) HCl ; d = 1.25 g/L , $w_1 = 29.2g$, $V = \frac{w}{d} = \frac{100}{1.25 \times 1000} L$

$W = 100g$, $M = \frac{W_1}{M_1 \times V}$, $M_1 = 36.5 g/mol$, $d = 1.25 g/ml$

$M_1V_1 = M_2V_2$, $1000 \times 10^{-2} \times V_1 = 0.4 \times 200$, $10 \times V_1 = 80$, $V_1 = 8ml$

40. An organic compound undergoes first-order decomposition. The time taken for its decomposition to 1/8 and 1/10 of its initial concentration are $t_{1/8}$ and $t_{1/10}$ respectively. What is the value of

$$\frac{[t_{1/8}]}{[t_{1/10}]} \text{x}10? \ (take \log_{10} 2 = 0.3)$$

40.(9) $t_{1/8} \to C = \frac{1}{8}C_0$, $t_{1/10} \to C = \frac{1}{10}C_0$, $\dfrac{t_{1/8} = \frac{2.303}{K}\log\frac{C_0 \times 8}{C_0}}{t_{1/10} = \frac{2.303}{K}\log\frac{C_0}{C_0}\times 10}$, $\quad \frac{t_{1/8}}{t_{1/10}} = \frac{\log 8}{1}$

$\frac{t_{1/8}}{t_{1/10}} = 3 \times 0.3 = 0.9$, $\quad \frac{t_{1/8}}{t_{1/10}} \times 10 = 9$

PART III : MATHEAMTICS

SECTION I : Single Correct Answer Type

This section contains 10 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

41. The total number of ways in which 5 balls of different colours can be distributed among 3 persons so that each person gets at least one ball is

(A) 75 (B) 150 (C) 210 (D) 243

41.(B) 5 balls $\Rightarrow$ 3 pesons $\Rightarrow$ $1,1,3$ / $2,2,1$

number of ways $= \left(\frac{5!}{1!1!3!2!} + \frac{5!}{2!2!1!2!}\right)\times 3!$ $= (10+15)\times 6 = 150$

42. Let $f(x) = \begin{cases} x^2\left|\cos\frac{\pi}{x}\right|, x \neq 0 \\ 0 \qquad\qquad , x = 0 \end{cases}, x \in IR$ then f is

(A) differentiable both at $x = 0$ and at $x = 2$

(B) differentiable at $x = 0$ but not differentiable at $x = 2$

(C) not differentiable at $x = 0$ but differentiable at $x = 2$

(D) differentiable neither at $x = 0$ nor at $x = 2$

42.(B) $f(x) = \begin{cases} x^2\left|\cos\frac{\pi}{x}\right|, \ x \neq 0 \\ 0 \qquad\qquad , x = 0 \end{cases}$

At $x = 0$

L.H.D. $= \lim_{h\to 0^+} \dfrac{(0-h)^2\left|\cos\frac{\pi}{h}\right| = 0}{(-h)}$ $= \lim_{h\to 0^+}(-h)\left|\cos\frac{\pi}{h}\right| = 0$

R.H.D. $= \lim_{h\to 0^+} \dfrac{(0-h)^2\left|\cos\frac{\pi}{h}\right| - 0}{(h)} = 0$

L.H.D. = R.H.D. $\Rightarrow f(x)$ is differentiable at $x = 0$

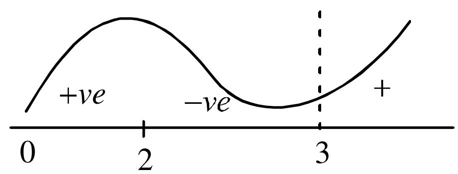
but $f(x)$ is not differentiable at $x=2$

43. The function $f:[0,3]\to[1,29]$ defined by $f(x)=2x^3-15x^2+36x+1$, is

(A) one- one and onto (B) onto but not one - one

(C) one - one but not onto (D) neither one - one nor onto

43.(B) $f(x)=2x^3-15x^2+36x+1$



$f'(x)=6x^2-30x+36$

$\Rightarrow 6(x^2-5x+6)\Rightarrow 6(x-2)(x-3)$

for max $\Rightarrow 0,2,3$

$f(0)=1$

$f(2)=16-60+72+1=29$

$f(3)=28$

$\therefore f(x)$ is onto as range $=[1,29]$ & Co- domain $=[1,29]$ (B) onto but not one - one

44. If $\lim\limits_{x\to\infty}\left(\dfrac{x^2+x+1}{x+1}-ax-b\right)=4$ then

(A) $a=1,b=4$ (B) $a=1,b=-4$ (C) $a=2,b=-3$ (D) $a=2,b=3$

44. (B) $\lim\limits_{x\to\infty}\left(\dfrac{x^2+x+1}{x+1}-ax-b\right)=4$ $\Rightarrow \lim\limits_{x\to\infty}\dfrac{x^2+x+1-ax^2-ax-bx-b}{x+1}=4$

$\Rightarrow \lim\limits_{x\to\infty}\dfrac{(1-a)x^2+x(1-a-b)+(1-b)}{x+1}=4$. Since limit existe

therefore $1-a=0\Rightarrow a=1$

$\Rightarrow \lim\limits_{x\to\infty}\dfrac{x(-b)+(1-b)}{x+1}=4$ $\Rightarrow \dfrac{-b}{1}=4\Rightarrow b=-4$

45. Let z be a complex number such that the imaginary part of z is nonzero and $a=z^2+z+1$ is real. Then a cannot take the value

(A) -1 (B) $\dfrac{1}{3}$ (C) $\dfrac{1}{2}$ (D) $\dfrac{3}{4}$

45.(D) $a=\left(z+\dfrac{1}{2}\right)^2+\dfrac{3}{4}$ when $z=-\dfrac{1}{2},a=\dfrac{3}{4}$ but z must have imaginary part non - zero $\therefore z\neq-\dfrac{1}{2}\therefore a\neq\dfrac{3}{4}$

46. The ellipse $E_1:\dfrac{x^2}{9}+\dfrac{y^2}{4}=1$ is inscribed in a rectangle R whose sides are parallel to the coordinate axes. Another ellipse E_2 passing through the point $(0,4)$ circumscribes the rectangle R. The eccentricity of the ellipse E_2 is

(A) $\dfrac{\sqrt{2}}{2}$ (B) $\dfrac{\sqrt{3}}{2}$ (C) $\dfrac{1}{2}$ (D) $\dfrac{3}{4}$

46.(C) $E_2: \dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}$ ....(i)

equation (i) passes through $(3,2)$

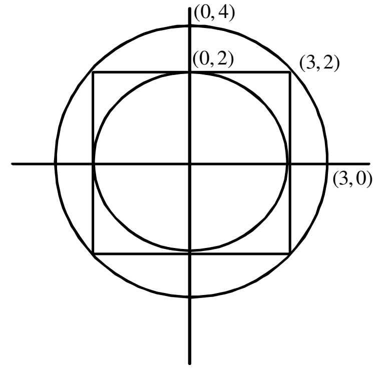
$\therefore \dfrac{9}{a^2}+\dfrac{4}{b^2}=1$

$(0,4)$ passes through equation (i)

$\therefore \dfrac{0}{a^2}+\dfrac{16}{b^2}=1$

$b^2=16, a^2=\dfrac{9\times 4}{3}=12$

$e^2=1-\dfrac{12}{16}\Rightarrow e^2=\dfrac{4}{16}\therefore e=\dfrac{1}{2}$



47. Let $P[a_{ij}]$ be a 3×3 matrix and let $Q=[b_{ij}]$, where $b_{ij}=2^{i+j}a_{ij}$ for $1\le i, j\le 3$. If the determinant of P is 2, then the determinant of the matrix Q is

(A) 2^{10} (B) 2^{11} (C) 2^{12} (D) 2^{13}

47.(D) $|P|=\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33}\end{vmatrix}$

$$|Q|=\begin{vmatrix} 2^2a_{11} & 2^3a_{12} & 2^4a_{13} \\ 2^3a_{21} & 2^4a_{22} & 2^5a_{23} \\ 2^4a_{31} & 2^5a_{32} & 2^6a_{33}\end{vmatrix}=2^2.2^3.2^4\begin{vmatrix} a_{11} & 2a_{12} & 2^2a_{13} \\ a_{21} & 2a_{22} & 2^2a_{23} \\ a_{31} & 2a_{32} & 2^2a_{33}\end{vmatrix}=2^2.2^3.2^4.2.2^2\,|P|=2^{12}\,|p|$$

48. The integral $\int \dfrac{\sec^2 x}{(\sec x+\tan x)^{9/2}}dx$ equals (for some arbitrary constant K)

(A) $-\dfrac{1}{(\sec x+\tan x)^{11/2}}\left\{\dfrac{1}{11}-\dfrac{1}{7}(\sec x+\tan x)^2\right\}+K$

(B) $\dfrac{1}{(\sec x+\tan x)^{11/2}}\left\{\dfrac{1}{11}-\dfrac{1}{7}(\sec x+\tan x)^2\right\}+K$

(C) $-\dfrac{1}{(\sec x+\tan x)^{11/2}}\left\{\dfrac{1}{11}+\dfrac{1}{7}(\sec x+\tan x)^2\right\}+K$

(D) $\dfrac{1}{(\sec x+\tan x)^{11/2}}\left\{\dfrac{1}{11}+\dfrac{1}{7}(\sec x+\tan x)^2\right\}+K$

48.(C) Put $(\sec x+\tan x)=t, \sec-\tan x=\dfrac{1}{t}$ $\Rightarrow 2\sec x=t+\dfrac{1}{t}\Rightarrow \sec x=\dfrac{t^2+1}{2t}$

$\sec(\sec x+\tan x)dx=dt$

$$\therefore \int \frac{\sec^2 x}{(\sec x+\tan x)^{9/2}} \frac{(\sec x+\tan x)}{(\sec x+\tan x)} dx$$

$$\int \frac{(\sec x)(\sec x+\tan x)\sec x\, dx}{(\sec x+\tan x)^{\frac{11}{2}}} \Rightarrow \int \frac{(\sec x)dt}{(t)^{11/2}} = \int \frac{\frac{t^2+1}{2t}}{t^{11/2}} dt$$

$$= \frac{1}{2}\int \frac{dt}{t^{13/2}} + \frac{1}{2}\int \frac{t^2}{t^{13/2}} dt \qquad = \frac{1}{2}\left[\frac{t^{-11/2}}{-11/2}\right] + \frac{1}{2}\int t^{(2-13/2)} dt \qquad = \left(\frac{1}{t^{11/2}}\right)\left(\frac{-1}{11}\right) + \frac{1}{2}\left(\frac{t^{-9/2+1}}{-7/2}\right) + C$$

$$= \left(\frac{-1}{11}\right)\left(\frac{1}{t^{11/2}}\right) + \left(\frac{1}{7}\right)\left(\frac{1}{t^{7/2}}\right) + C = -\left[\frac{1}{(\sec x+\tan x)^{11/2}}\right]\left[\frac{1}{11} + \frac{1}{7}(\sec x+\tan x)^2\right] + C$$

49. The point P is the intersection of the straight line joining the point $Q(2,3,5)$ and $R(1,-1,4)$ with the plane $5x-4y-z=1$. If S is the foot of the perpendicular drawn from the point $T(2,1,4)$ to QR then the length of the line segment PS is

(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$ (C) 2 (D) $2\sqrt{2}$

49.(A) Direction ratios of $QR(1,4,1)$

P divides QR in the ratio $k:1$ where $\frac{k}{1} = \frac{-(5\times 2-4\times 3-5-1)}{(5\times 1-4\times(-1)-4-1)} = \frac{2}{1}$

$\therefore P$ is

$$\left(\frac{2\times 1+1\times 2}{2+1}, \frac{2\times(-1)+1\times 3}{2+1}, \frac{2\times 4+1\times 5}{2+1}\right)$$

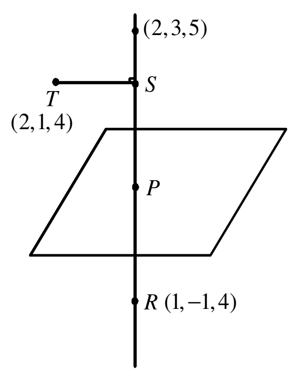
$$\left(\frac{4}{4}, \frac{1}{3}, \frac{13}{3}\right)$$

equation of PQ

$$\frac{x-1}{1} = \frac{y+1}{4} = \frac{z-4}{1} = k$$

$\therefore S$ can be $(k+1, 4k-1, k+4)$

$TS \perp PQ$



$\therefore (k+1-2)\times 1+(4k-1-1)\times 4+(k+4-4)\times 1=0$

$k-1+16k-8+k=0$

$\Rightarrow 18k=9\Rightarrow k=\frac{1}{2}$

$\therefore S=\left(\frac{3}{2},1,\frac{9}{2}\right)$

$PS=\sqrt{\left(\frac{3}{2}-\frac{4}{3}\right)^2+\left(1-\frac{1}{3}\right)^2+\left(\frac{9}{2}-\frac{13}{3}\right)^2}=\sqrt{\frac{1}{36}+\frac{4}{9}+\frac{1}{36}}=\frac{1}{\sqrt{2}}$

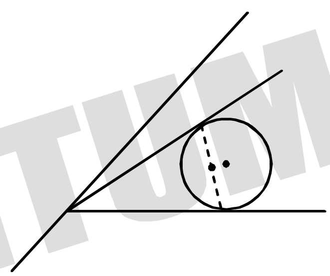
50. The locus of the mid-point of the chord of contact of tangents drawn from points lying on the straight line $4x-5y=20$ to the circle $x^2+y^2=9$ is

(A) $20(x^2+y^2)-36x+45y=0$
(B) $20(x^2+y^2)+36x-45y=0$
(C) $36(x^2+y^2)-20x+45y=0$
(D) $36(x^2+y^2)+20x-45y=0$

50.(A) $4x-5y=20$

Let a point on the line be

$P\left(\frac{5t+20}{4},t\right)$ let the mid point of chord of contact be (x',y')



equation of chord $T=S_1$

$xx'+yy'-9=x'^2+y'^2-9$ ....(i)

$xx'+yy'=x'^2+y'^2$ & equation of chord of contact of tangents from P be $T=0$ $x\left(\frac{5t+20}{4}\right)+yt-9=0$

......(ii)

(i) & (ii) are identical

$\therefore \frac{x'}{\frac{5t+20}{4}}=\frac{y'}{t}=\frac{x'^2+y'^2}{9}$ $\Rightarrow tx'=y'\left(\frac{5t+20}{4}\right)$

$\therefore t=\frac{20y'}{4x'-5y'}=\frac{9y'}{x'^2+y'^2}$

$20(x'^2+y'^2)-36x'+45y'$

SECTION II : Multiple Correct Answer(s) Type

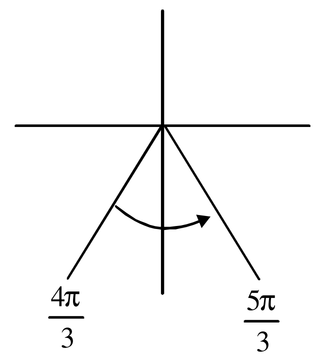
This section contains 5 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

51. Let $\theta,\phi\in[0,2\pi]$ be such that $2\cos\theta(1-\sin\phi)=\sin^2\theta\left(\tan\frac{\theta}{2}+\cot\frac{\theta}{2}\right)\cos\phi-1$, $\tan(2\pi-\theta)>0$ and

$-1 < \sin\theta < -\frac{\sqrt{3}}{2}$. Then ϕ cannot satisfy

(A) $0 < \phi < \frac{\pi}{2}$ (B) $\frac{\pi}{2} < \phi < \frac{4\pi}{3}$ (C) $\frac{4\pi}{3} < \phi < \frac{3\pi}{2}$ (D) $\frac{3\pi}{2} < \phi < 2\pi$

51.(A,C,D) $-1 < \sin\theta < -\frac{\sqrt{3}}{2}$ ...(i)



$$2\cos\theta(1-\sin\phi) = \frac{2\sin^2\theta}{2\cos\theta/2.\sin\theta/2}\cos\phi - 1$$

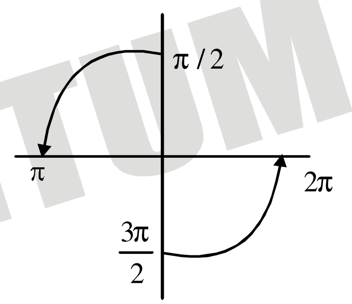
$$\Rightarrow 2\cos\theta(1-\sin\phi) = 2\sin\theta\cos\phi - 1$$

$$\Rightarrow 2\sin\theta\cos\phi + 2\cos\theta\sin\phi = 1 + 2\cos\theta$$

$$\sin\phi\cos\phi + \cos\theta\sin\phi = \frac{1}{2} + \cos\theta$$

$\frac{1}{2} < \sin(\theta+\phi) < 1$ from (i)

$\Rightarrow \frac{13\pi}{6} < \theta + \phi < \frac{17\pi}{6}$ and $\frac{-5\pi}{3} < -\theta < \frac{-3\pi}{2} \Rightarrow \frac{\pi}{2} < \phi < \frac{4\pi}{3}$



$\tan(2\pi - \theta) > 0$ ...(ii)

$\therefore$ (i) & (ii) $\Rightarrow \phi \in \left(\frac{3\pi}{2}, \frac{5\pi}{3}\right)$

$\frac{3\pi}{2} < \theta < \frac{5\pi}{3}$, $0 < \phi < 2\pi \Rightarrow \frac{3\pi}{2} < \theta + \phi < \frac{11\pi}{3}$

52. Let S be the area of the region enclosed by $y = e^{-x^2}, y = 0, x = 0$ and $x = 1$. Then

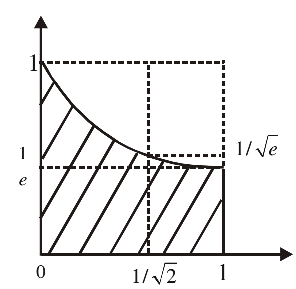
(A) $S \ge \frac{1}{e}$ (B) $S \ge 1 - \frac{1}{e}$ (C) $S \le \frac{1}{4}\left(1 + \frac{1}{\sqrt{e}}\right)$ (D) $S \le \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{e}}\left(1 - \frac{1}{\sqrt{2}}\right)$

52.(A,B,D)

$y = e^{-x^2}$

$y^1 = \left(e^{-x^2}\right)(-2x)$

$\therefore$ function is decreasing



$S \ge \left(\frac{1}{e} \text{ x } 1\right)$ and $\le \frac{1}{\sqrt{2}} \cdot 1 + \left(1 - \frac{1}{\sqrt{2}}\right)\frac{1}{\sqrt{e}}$

$\because e^{-x^2} \ge e^{-x} \forall x \in [0,1]$ $\Rightarrow S \ge \int_0^1 e^{-x}dx \Rightarrow S \ge 1 - \frac{1}{e}$

53. A ship is fitted with three engines E_1, E_2 and E_3. The engines function independently of each other with respective probabilities $\frac{1}{2}, \frac{1}{4}$ and $\frac{1}{4}$. For the ship to be operational at least two of its engines must function. Let X denote the event that the ship is operational and let X_1, X_2 and X_3 denote respectively the events that the engines E_1, E_2 and E_3 are functioning. Which of the following is (are) true?

(A) $P\left[X_1^c \middle| X\right] = \frac{3}{16}$

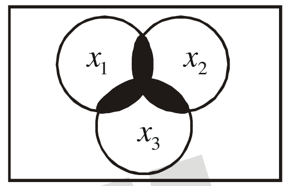
(B) P [Exactly two engines of the ship are functioning $|X$] $= \frac{7}{8}$

(C) $P\left[X \middle| X_2\right] = \frac{5}{16}$

(D) $P\left[X \middle| X_1\right] = \frac{7}{16}$

53.(B,D)

shaded region represent X



$$P(x_1) = \frac{1}{2} P(x_2) = \frac{1}{4} P(x_3) = \frac{1}{4}$$

$$P(X) = P(X_1 \cap X_2 \cap X_3{}^c) + P(X_1 \cap X_2{}^c \cap X_3)$$

$$+P(X_1{}^c \cap X_2 \cap X_3) + P(X_1 \cap X_2 \cap X_3) = \left(\frac{1}{2}\right)\left(\frac{1}{4}\right)\left(\frac{3}{4}\right) + \left(\frac{1}{2}\right)\left(\frac{3}{4}\right)\left(\frac{1}{4}\right)$$

$$+\left(\frac{1}{2}\right)\left(\frac{1}{4}\right)\left(\frac{1}{4}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{4}\right)\left(\frac{1}{4}\right)$$

$$P\left(\frac{X_1{}^c}{X}\right) = \frac{P\left(X_1 \cap X\right)}{P(X)} = \frac{P(X_2 \cap X_3) - P(X_1 \cap X_2 \cap X_3)}{P(X)} = \frac{1}{8}$$

Let $T \Rightarrow$ exactly two engines are functioning

then $P\left(\frac{T}{X}\right) = \frac{P(T \cap X)}{P(X)} = \frac{P(X_1 \cap X_2) + P(X_2 \cap X_3)}{P(X)}$

$$\frac{+P(X_1 \cap X_3) - 3(X_1 \cap X_2 \cap X_3)}{P(X)} = \frac{\frac{1}{8} + \frac{1}{16} + \frac{1}{8} - \frac{3}{32}}{\frac{1}{4}} = \frac{\frac{7}{32}}{\frac{1}{4}} = \frac{7}{8}$$

$$P\left(\frac{X}{X_2}\right) = \frac{P(X \cap X_2)}{P(X_2)} = \frac{P(X_1 \cap X_2) + P(X_2 \cap X_3) - P(X_1 \cap X_2 \cap X_3)}{P(X_2)}$$

$$= \frac{P(X_1)P(X_2) + P(X_2)P(X_3) - P(X_1)P(X_2)P(X_3)}{P(X_2)}$$

$$=\frac{\left(\frac{1}{2}\right)\left(\frac{1}{4}\right)+\left(\frac{1}{4}\right)\left(\frac{1}{4}\right)-\left(\frac{1}{2}\right)\left(\frac{1}{4}\right)\left(\frac{1}{4}\right)}{\left(\frac{1}{4}\right)}$$

$$=\frac{\frac{1}{8}+\frac{1}{16}-\frac{1}{32}}{\frac{1}{4}}=\frac{\frac{5}{32}}{\frac{1}{4}}=\frac{5}{8}$$

$$P\left(\frac{X}{X_1}\right)=\frac{P(X\cap X_1)}{P(X_1)}=\frac{P(X_1\cap X_2)+P(X_1\cap X_3)-P(X_1\cap X_2\cap X_3)}{P(X_1)}=\left(\frac{7}{16}\right)$$

54. Tangents are drawn to the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$,parallel to the straight line $2x-y=1$. The points of contact of the tangents on the hyperbola are

(A) $\left(\frac{9}{2\sqrt{2}},\frac{1}{\sqrt{2}}\right)$ (B) $\left(-\frac{9}{2\sqrt{2}},-\frac{1}{\sqrt{2}}\right)$ (C) $\left(3\sqrt{3},-2\sqrt{2}\right)$ (D) $\left(-3\sqrt{3},2\sqrt{2}\right)$

54.(A,B)

Let $P(3\sec\theta,2\tan\theta)$ be a point on hyperbola tangent at p

$$\frac{x\sec\theta}{3}-\frac{y\tan\theta}{2}=1$$

given $-\frac{\sec\theta}{3}\times\frac{-2}{\tan\theta}=2$

$$\sec\theta=3\tan\theta\Rightarrow\sin\theta=\frac{1}{3}$$

$$\therefore\sec\theta=\pm\frac{3}{2\sqrt{2}}\tan\theta=\pm\frac{1}{2\sqrt{2}}$$

point of contact will be is 1st and 3rd quadrant and will be

$$(a\sec\theta,b\tan\theta)=\left(\frac{9}{2\sqrt{2}},\frac{1}{\sqrt{2}}\right),\left(\frac{-9}{2\sqrt{2}},\frac{-1}{\sqrt{2}}\right)$$

55. If $y(x)$ satisfies the differential equation $y'-y\tan x=2x\sec x$ and $y(0)=0$, then

(A) $y\left(\frac{\pi}{4}\right)=\frac{\pi^2}{8\sqrt{2}}$ (B) $y'\left(\frac{\pi}{4}\right)=\frac{\pi^2}{18}$ (C) $y\left(\frac{\pi}{3}\right)=\frac{\pi^2}{9}$ (D) $y'\left(\frac{\pi}{3}\right)=\frac{4\pi}{3}+\frac{2\pi^2}{3\sqrt{3}}$

55.(A,D) Differentiable equation

$\frac{dy}{dx}-y\tan x=2x\sec x$ it is form $\frac{dy}{dx}+py=q$

$$I.F.=e^{\int pdx}=e^{-\int\tan x\,dx}=1/\sec x$$

$$y(I.F.=)\int Q(I.F.)dx + C \Rightarrow y\left(\frac{1}{\sec x}\right) = 2\int x\,dx + C \Rightarrow y/(\sec x) = \frac{2x^2}{2} + C.$$

Since $y(0) = 0 \Rightarrow c = 0$

$$y = x^2 \sec x \Rightarrow y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{16}\sqrt{2} = \frac{\pi^2}{8\sqrt{2}}$$

$$y' = 2x\sec x + x^2 \sec x \tan x$$

$$y'\left(\frac{\pi}{3}\right) = \frac{2\pi}{3}(2) + \frac{\pi^2}{9}(2)\left(\sqrt{3}\right) = \frac{4\pi}{3} + \frac{2\pi^2\sqrt{3}}{9} = \frac{4\pi}{3} + \frac{2\pi^2}{3\sqrt{3}}$$

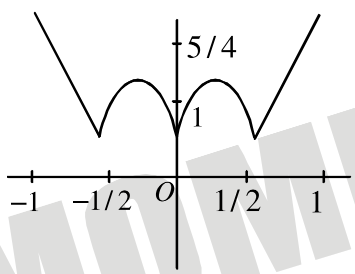
SECTION III : Integer Answer Type

This section contains 5 questions. The answer to each question is a single digit integer, ranging from 0 to 9 (both inclusive).

56. Let $f : IR \to IR$ be defined as $f(x) = |x| + |x^2 - 1|$. The total number of points at which f attains either a local maximum or a local minimum is

56.(5)

$|x| + |x^2 - 1|$



$x < -1$

$$f(x) = -x + x^2 - 1 = \left(x - \frac{1}{2}\right)^2 - \frac{5}{4}$$

$-1 \le x < 0$

$$f(x) = -x - x^2 + 1 = \frac{5}{4} - \left(x + \frac{1}{2}\right)^2$$

$$0 \le x < 1, f(x) = x - x^2 + 1 = \frac{5}{4} - (x - 1/2)^2$$

$$x \ge 1, f(x) = x + x^2 - 1 = \left(x + \frac{1}{2}\right)^2 - \frac{5}{4}$$

57. The value of $6 + \log_{\frac{3}{2}}\left(\frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}.....}}}\right)$ is

57.(4) $6+\log 3/2\left(\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}}}\right)$

Now $\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}.....}}=x$

$\frac{1}{18}\left(4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}....}\right)=x^2$

$\frac{4}{18}-\frac{1}{18}x=x^2$

$4-x=18x^2$

$18x^2+x-4=0$

$x=\frac{\pm\sqrt{1-4(-4)(18)}}{2(18)}=\frac{\pm\sqrt{289}}{36}=\frac{1+17}{36}=\frac{16}{36}=\frac{4}{9}\Rightarrow 6+\log_{3/2}\left(\frac{4}{9}\right)=6+\log_{(3/2)}{(2/3)^2}$

$=6+\log_{(3/2)}{(2/3)^{-2}}\quad =6-2=4$

58. Let $p(x)$ be a real polynomial of least degree which has a local maximum at $x=1$ and a local minimum at $x=3$. If $p(1)=6$ and $p(3)=2$, then $p'(0)$ is

58.(9) $P^1(x)=a(x-1)(x-3)$

$P(x)=a\left[\frac{x^3}{3}-2x^2+3x\right]+C$

$P(1)=6\Rightarrow a\left(\frac{1}{3}-2+3\right)+C\quad\Rightarrow 6=\frac{4}{3}a+C$

$p(3)=2\Rightarrow 2=a(9-18+9)+c\quad\Rightarrow C=2$

$So, 6=\frac{4}{3}\;a+2\quad\Rightarrow a=3\;\Rightarrow P\;(0)=3\;(0-1)\;(0-3)\;=9$

59. If $\vec{a},\vec{b}$ and $\vec{c}$ are unit vectors satisfying $\left|\vec{a}-\vec{b}\right|^2+\left|\vec{b}-\vec{c}\right|^2+\left|\vec{c}-\vec{a}\right|^2=9$, then $\left|2\vec{a}+5\vec{b}+5\vec{c}\right|$ is

59.(3) $\vec{a}^2+\vec{b}^2-2\vec{a}.\vec{b}+\vec{b}^2+\vec{c}^2-2\vec{b}.\vec{c}\;\;+\vec{c}^2+\vec{a}^2-2\vec{c}.\vec{a}=9$

$6-2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})=9$

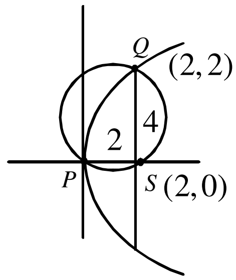
$\therefore(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})=(-3/2)$

only when $(\vec{a}+\vec{b}+\vec{c}=0)$

Now, $\left|(2\vec{a}+5\vec{b}+5\vec{c})\right|\;\Rightarrow\sqrt{(2\vec{a}+5(\vec{b}+\vec{c}))^2}\;\Rightarrow\sqrt{(2\vec{a}-5\vec{a})^2}=\sqrt{(-3\vec{a})^2}=\sqrt{9a^2}=3$

60. Let S be the focus of the parabola $y^2 = 8x$ and let PQ be the common chord of the circle $x^2 + y^2 - 2x - 4y = 0$ and the given parabola. The area of the triangle PQS is

60.(4) $y^2 = 8x$ & $x^2 + y^2 - 2x - 4y = 0$



Focus $S = (2,0)$ ends of latus rectum are $(2,4)$ & $(2-4)$ circle & parabola meet at $(0,0)$ say P & $(2,4)$, say Q $\Delta PQS = \frac{1}{2} \times 2 \times 4 = 4$