AIEEE 2010 PAPER -1 : CHEMISTRY, MATHEMATICS & PHYSICS

Test Booklet Code

Do not open this test booklet until you are asked to do so.

Read carefully the Instructions on the back cover fo this Test Booklet.

B

Important Instructions :

1. Immediately fill in the particulars on this page of the test booklet with Blue ball point pen. Use of pencil is strictly prohibited.
2. The Answer sheet is kept inside this test booklet. When you are directed to open the Test booklet, take out the answer sheet and fill in the particulars carefully.
3. The test is of **3 hours** duration.
4. The test booklet consists of **90** questions. The maximum marks are **432**.
5. There are **three** parts in the question paper. The distribution of marks subjectwise in each part is as under for each correct response.

   Part A - **Chemistry(114 marks)**- Question No. 4 to 16 and 20 to 30 consists of **FOUR(4)** marks each and Question No. 1 to 3 and 17 to 19 consist of **EIGHT(8)** marks each for each correct response.

   Part B - **Mathematics(114 marks)**- Question No. 34 to 42, 46 to 60 consist of **FOUR(4)** marks each and Questions No. 31 to 33 and 43 to 45 consist of **EIGHT(8)** marks each for each correct response.

   Part C- **Physics(144 marks)**-Question No. 65 to 68 and 71 to 90 consist of **FOUR(4)** marks each and Questions No. 61 to 64 and 69 to 70 consist of **EIGHT(8)** marks each for each correct response.

6. Candidates will be awarded marks as stated above in Instruction No. 5 for correct response of each question. ¼(onr fourth) marks will be deducted for indicating incorrect response of each question. No **deduction** from the total score will be made **if no response** is indicated for an item in the Answer sheet.
7. Use **Blue/Black point pen only** for writing particular/marking responses on **Side-1** and **Sind-2** of the answer sheet. **Use of pencil is strictly prohibited**.
8. No candidate is allowed to carry any textual materis printed or written, bits of papers pager, mobile, phone, any electronci device, etc., except the Abmit card inside the examination hall/room.
9. Rough work is to be done on the space provided for this purpose in the test booklet only. This space is given at the bottom of each page and in 2 pages (Pages 38-39) at the end of the booklet.
10. On completion of the test, the candidate must hand over the Answer Sheet to the invigilator on duty in the Room/Hall. ***However the candidates are allowed to take away this test booklet with them.***
11. The CODE for this booklet is **B**. Make sure that the CODE printed on **Side-2** of the answer sheet is the same as that on this booklet. In case of discrepancy, the candidate should immediately report the matter to the invigilator for replacement of both the test boolket and the answer sheet.
12. ***Do not fold or make any stray marks on the Answer Sheet.***

PART - "A" PHYSICS

Directions :

Questions number 1 - 3 are based on the following paragraph

An initially parallel cylindrical beam travels in a medium of refractive index $\mu(I)=\mu_0+\mu_2 I$, where μ_0 and μ_2 are positive constants and I is the intensity of the light beam. The intensity of the beam is decreasing with increasing radius.

1. The initial shape of the wavefront of the beam is

(a) planar

(b) convex

(c) concave

(d) convex near the axis and concave near the periphery

Sol. **(a)**

2. The speed of light in the medium is

(a) maximum on the axis of the beam

(b) minimum on the axis of the beam

(c) the same eveywhere in the beam

(d) directly proportional to the intensity I

Sol. **(c)**

3. As the beam enters the medium, it will

(a) travel as a cylindrical beam

(b) diverge

(c) converge

(d) diverge near the axis and converge near the periphery

Sol. **(a)**

Directions :

Questions number 4-5 are based on the following paragraph.

A necleus of mass $M+\Delta m$ is at rest and decays into two daughter nuclei of equal mass $\frac{M}{2}$ each. Speed of light is c.

4. The speed of daughter nuclei is

(a) $c\sqrt{\frac{\Delta m}{M+\Delta m}}$ (b) $c\frac{\Delta m}{M+\Delta m}$

(c) $c\sqrt{\frac{2\Delta m}{M}}$ (d) $c\sqrt{\frac{\Delta m}{M}}$

Sol. **(c)**

Speed of daughter Nuclei

$$\frac{1}{2}\left(\frac{M}{2}\right)V^2+\frac{1}{2}\left(\frac{M}{2}\right)V^2=\Delta mc^2$$

$$\frac{1}{2}M\times V^2=\Delta mc^2$$

$$V=\sqrt{\frac{2\Delta mc^2}{M}}$$

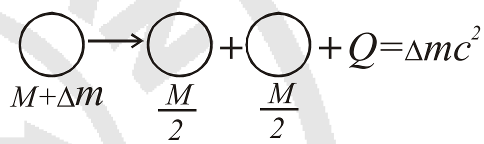
$$V=\sqrt{\frac{2\Delta m}{M}}\cdot c$$

5. The binding energy per nucleon for the parent nucleus is E_1 and that for the daughter nuclei is E_2. Then

(a) $E_1=2E_2$ (b) $E_2=2E_1$

(c) $E_1>E_2$ (d) $E_2>E_1$

Sol. **(d)**



$E_1=$ B.E. per nucleon of Parent Nuclei

$E_2=$ B.E. per nucleon of daughter Nuclei

$E_2>E_1$

Directions :

Questions number 6-7 contain Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.

6. **Statement - 1 :**

When ultraviolet light is incident on a photocell, its stopping potential is V_0 and the maximum kinetic energy of the photoelectrons is $K_{\max}$. When the ultraviolet light is replaced by X-rays, both V_0 and $K_{\max}$ increase.

Statement - 2 :

Photoelectrons are emitted with speeds ranging from zero to a maximum value because of the range of frequencies present in the incident light.

(a) Statement - 1 is true, Statement - 2 is false.

(b) Statement - 1 is true, Statement - 2 is true; Statement - 2 is the correct explanation of Statement -1.

(c) Statement - 1 is true, Statement - 2 is true; Statement - 2 is not correct explanation of Statement -1.

(d) Statement - 1 is false, Statement - 2 is true.

Sol. **(a)**

7. **Statement - 1 :**

Two particles moving in the same direction do not lose all their energy in a completely inelastic collision.

Statement - 2 :

Principle of conservation of momentum holds true for all kinds of collisions.

(a) Statement - 1 is true, Statement - 2 is false.

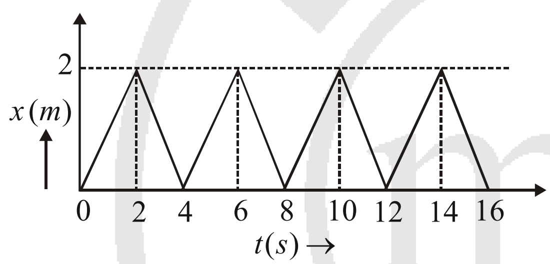
(b) Statement - 1 is true, Statement - 2 is true; Statement - 2 is the correct explanation of Statement -1.

(c) Statement - 1 is true, Statement - 2 is true; Statement - 2 is not correct explanation of Statement -1.

(d) Statement - 1 is false, Statement - 2 is true.

Sol. **(d)**

8. The figure shows the position - time (x-t) graph of one-dimentsional motion of a body of mass 0.4 kg. The magnitude of each impulse is



(a) 0.2 Ns (b) 0.4 Ns

(c) 0.8 Ns (d) 1.6 Ns

Sol. **(c)**

$m = 0.4\,kg$

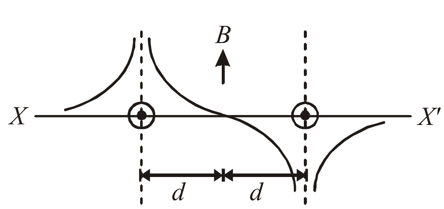
$I = \Delta \rho$

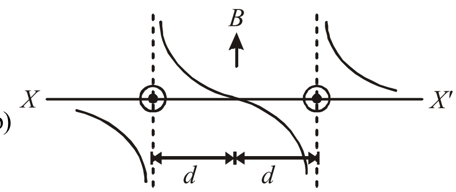
$= m\cdot(V_t - V_i)$

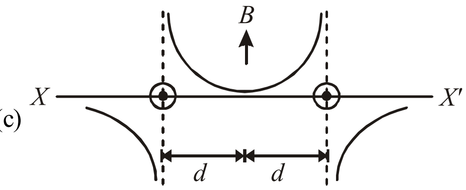
$= \left|0.4\times\left(\frac{2}{2}-\left(\frac{-2}{2}\right)\right)\right|$

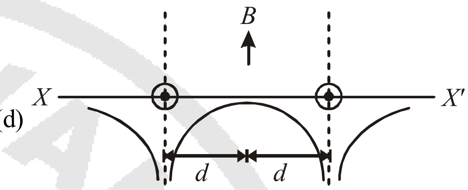
$= 0.4\times 2$ $= 0.8\,N-s$

9. Two long parallel wires are at a distance 2d apart. They carry steady equal currents flowing out of the plane of the paper as shown. The variation of te magnetic field B along the line XX' is give by

(a)


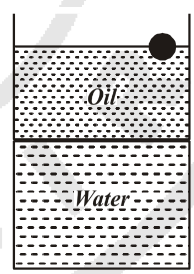
(b)


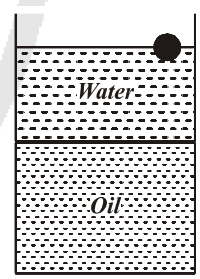
(c)


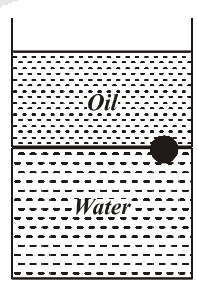
(d)


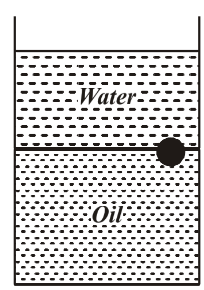
Sol. **(b)**

10. A ball is made of a material of density ρ where $\rho_{oil} < \rho < \rho_{water}$ wih ρ_{oil} and ρ_{water} representing the densities of oil and water, respectively. The oil and water are immiscible. If the above ball is in equilibrium in a mixture of this oil and water, which of the folowing pictures represents its equilibrium position ?

(a)


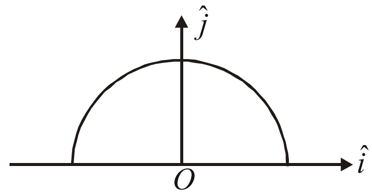
(b)


(c)


(d)


Sol.(c)

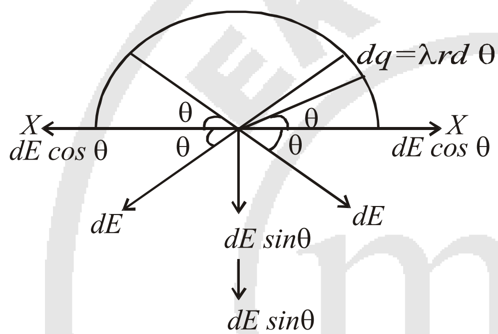
11. A thin semi-circular ring of radius r has a positive charge q distributed uniformly over it. The net field $\vec{E}$ at the centre O is :



(a) $\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$ (b) $\frac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$

(c) $-\frac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$ (d) $-\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$

Sol.(d) $\lambda = \frac{q}{\pi r}$



$$E = \int d\varepsilon \sin\theta$$

$$= \int \frac{1}{4\pi\varepsilon_0}\frac{q}{\pi r}\frac{rd\theta}{r^2}\sin\theta$$

$$= \frac{q}{4\pi^2\varepsilon_0 r^2}\left[-\cos\theta\right]$$

$$= \frac{-q}{4\pi^2\varepsilon_0 r^2}\left[\cos\theta\right]_0^{\pi} \qquad = \frac{2q}{4\pi^2\varepsilon_0 r^2}$$

$$E = \frac{q}{2\pi^2\varepsilon_0 r^2}$$

$$\vec{E} = \frac{-q}{2\pi^2\varepsilon_0 r^2}\hat{j}$$

12. A diatomic ideal gas is used in a Carnot engine as the working substance. If during the adiabatic expansion part of the cycle the volume of the gas increases from V to $32V$, the efficiency of the engine is :

(a) 0.25 (b) 0.5

(c) 0.75 (d) 0.99

Sol. **(c)**

$PV^{\gamma} =$ constant

$\frac{nRT}{V}V^{\gamma} =$ constant $\gamma = 7/5$

$TV^{\gamma-1} =$ constant $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} = (32)^{\frac{7}{5}-1}$$

$$\frac{T_1}{T_2} = (32)^{\frac{2}{5}} = (2^5)^{\frac{2}{5}} = 2^2$$

$$\frac{T_1}{T_2} = 4 \Rightarrow \frac{T_2}{T_1} = \frac{1}{4}$$

$$\frac{T_2}{T_1} - 1 = \frac{1}{4} - 1$$

$$\frac{T_2 - T_1}{T_1} = \frac{1}{4} - 1 = \frac{-3}{4}$$

$$\eta = \frac{T_1 - T_2}{T_1} = \frac{3}{4} = 75\%$$

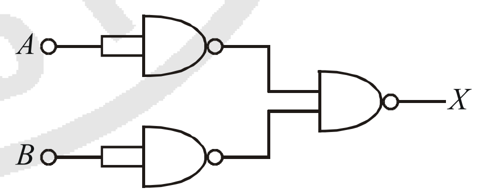
13. The respective number of significant figure for the numbers 23.023, 0.0003 and 2.1×10^{-3} are

(a) 4, 4, 2 (b) 5, 1, 2

(c) 5, 1, 5 (d) 5, 5, 2

Sol. **(b)**

14. The combination of gates shown below yields:



(a) NAND gate (b) OR gate

(c) NOT gate (d) XOR gate

Sol. **(b)**

$X = \overline{\overline{A}\cdot\overline{B}}$ $= \overline{\overline{A}} + \overline{\overline{B}}$

$= A + B$ $= OR$

15. If a source of power 4 kW produces 10^{20} photons/second, the radiation belongs to a part of the spectrum called

(a) γ-rays (b) X-rays

(c) ultraviolet rays (d) microwaves

Sol. **(b)**

$$4\times1000=\frac{10^{20}\times6.6\times10^{-34}\times3\times10^{8}}{\lambda}$$

$$\lambda=\frac{19.8\times10^{-6}}{4\times10^{3}}\qquad=4.95\times10^{-9}$$

$$=49.5\mathring{A}$$ X-ray

16. A radioactive nucleus (initial mass number A and atomic number Z) emits 3 $\alpha-$particles and 2 positrons. The ratio of number of neutrons to that of protons in the final nucleus will be

(a) $\frac{A-Z-4}{Z-2}$ (b) $\frac{A-Z-8}{Z-4}$

(c) $\frac{A-Z-4}{Z-8}$ (d) $\frac{A-Z-12}{Z-4}$

Sol. **(c)**

$${}_{Z}X^{A}\rightarrow{}_{z-8}Y^{A-12}+3\,{}_{2}\alpha^{4}+2\,{}_{H}\beta^{0}$$

$\downarrow$

Final Neutron

$$\frac{No.of\ Neutron}{No.of\ \Pr oton}=\frac{A-12-(z-8)}{Z-8}$$

$$=\frac{A-Z-4}{Z-8}$$

17. Let there be a spherically symmetric charge distribution with charge density varying as $\rho(r)=\rho_0\left(\frac{5}{4}-\frac{r}{R}\right)$ upto $r=R$, and $\rho(r)=0$ for $r>R$, where r is the distance from the origin. The electric field at a distance $r(r<R)$ from the origin is given by

(a) $\frac{\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4}-\frac{r}{R}\right)$ (b) $\frac{4\pi\rho_0 r}{3\varepsilon_0}\left(\frac{5}{3}-\frac{r}{R}\right)$

(c) $\frac{\rho_0 r}{4\varepsilon_0}\left(\frac{5}{3}-\frac{r}{R}\right)$ (d) $\frac{4\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4}-\frac{r}{R}\right)$

Sol.(c) $\rho=\rho_0\left(\frac{5}{4}-\frac{r}{R}\right),\ r\le R$

$$\rho=0,r>R\qquad\Rightarrow dq=\rho.dV$$

$$q=\int_0^r\rho_0\left(\frac{5}{4}-\frac{x}{R}\right).4\pi x^2dx$$

$$=4\pi\rho_0\left[\int_0^r\frac{5}{4}x^2dx-\frac{1}{R}\int_0^r x^3dx\right]$$

$$=4\pi\rho_0\left[\frac{5}{4}\left[\frac{x^3}{3}\right]_0^r-\frac{1}{R}\left[\frac{x^4}{4}\right]_0^r\right]$$

$$=4\pi\rho_0\left[\frac{5}{12}r^3-\frac{r^4}{4R}\right]$$

$$=\frac{4\pi\rho_0}{4}r^3\left[\frac{5}{3}-\frac{r}{R}\right]$$

$$q=\pi\rho_0 r^3\left[\frac{5}{3}-\frac{r}{R}\right]$$

$$E=\frac{1}{4\pi\in_0}\frac{q}{r^2}=\frac{1}{4\pi\in_0}\frac{\pi\rho_0 r^3}{r^2}\left[\frac{5}{3}-\frac{r}{R}\right]$$

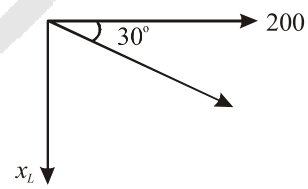
$$E=\frac{\rho_0 r}{4\in_0}\left[\frac{5}{3}-\frac{r}{R}\right]$$

18. In a series LCR circuit $R=200\Omega$ and the voltage and the frequency of the main supply is 220 V and 50 Hz respectively. On taking out the capacitance from the circuit the current lags behind the voltage by 30°. On taking out the inductor from the circuit the current leads the voltage by 30°. The power dissipated in the LCR circuit is :

(a) 242 W (b) 305 W

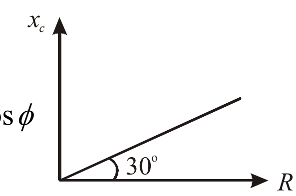
(c) 210 W (d) Zero W

Sol. **(a)**



$$\frac{1}{\sqrt3}=\frac{x_L}{200}$$

$$\frac{1}{\sqrt3}=\frac{x_c}{200}$$



$$\Rightarrow x_c=x_L$$

$$P_{av}=V_{rms}I_{rms}\cos\phi$$

$$=220\times\frac{220}{200}\times1$$

$$=\frac{220\times220}{200}=\frac{484}{2}=242$$

19. In the circuit shown below, the key K is closed at t = 0. The current through the battery is

(a) $\dfrac{V(R_1+R_2)}{R_1R_2}$ at $t=0$ and $\dfrac{V}{R_2}$ at $t=\infty$

(b) $\dfrac{V R_1R_2}{\sqrt{R_1^2+R_2^2}}$ at $t=0$ and $\dfrac{V}{R_2}$ at $t=\infty$

(c) $\dfrac{V}{R_2}$ at $t=0$ and $\dfrac{V(R_1+R_2)}{R_1 R_2}$ at $t=\infty$

(d) $\dfrac{V}{R_2}$ at $t=0$ and $\dfrac{V R_1 R_2}{\sqrt{R_1^{\,2}+R_2^{\,2}}}$ at $t=\infty$

Sol.(c) At $t=0,$ current in $ind=0$

$$I=\frac{v}{R_2}$$

at $t=\infty,$ of inductor $=0$

$$R_{eq}=\frac{R_1 R_2}{R_1+R_2}$$

$$I=\frac{v(R_1+R_2)}{R_1 R_2}$$

20. A particle is moving with velocity $\vec{v}=K(y\,\hat{i}+x\,\hat{j}),$ where K is a constant. The general equation for its path is :

(a) $y^2=x^2+$ constant

(b) $y=x^2+$ constant

(c) $y^2=x+$ constant

(d) $xy=$ constant

Sol.(a) $v=k(y\,\hat{i}+x\,\hat{j})$

$v_x=k\,y$

$v_y=k\,x$

$\Rightarrow \dfrac{v_y}{v_x}=\dfrac{x}{y}$ $\qquad\Rightarrow x\,v_x=y\,v_y$

$$\int \frac{xd_x}{dt}=\int \frac{yd_y}{dt}$$

$$\int xdx=\int ydy$$

$\Rightarrow \dfrac{x^2}{2}=\dfrac{y^2}{2}$ $\qquad\Rightarrow x^2=y^2+c$

$\Rightarrow y^2=x^2+c$

21. Let C be the capacitance of a capacitor discharging through a resistor R. Suppose t_1 is the time taken for the energy stored in the capacitor to reduce to half its initial value and t_2 is the time taken for the charge to reduce to one-fourth its initial value. Then the ratio t_1/t_2 will be:

(a) 2 (b) 1

(c) $1/2$ (d) $1/4$

Sol.(d) $U=\dfrac{Q^2}{2c}\propto Q^2$

$$\frac{U}{U/2}=\left(\frac{Q}{Q_1}\right)^2$$

$$2=\left(\frac{Q}{Q_1}\right)^2$$

$$\frac{Q}{Q_1}=\sqrt{2}$$

$\Rightarrow Q_0\,e^{-}t_1/\tau=\dfrac{Q_0}{\sqrt{2}}$ $\qquad\Rightarrow e\,t_1/\tau=\sqrt{2}$

$$\frac{t_1}{t}=ln\sqrt{2}$$

$$t_1=\ln\sqrt{2}.\tau$$

$$Q=\frac{Q_0}{4}$$

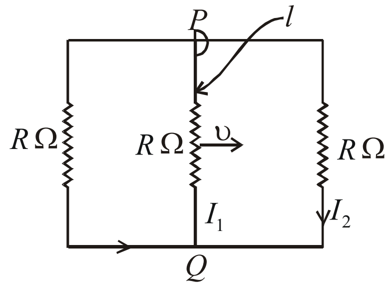
$$Q_0\,e^{-}t_2/\tau=\frac{Q_0}{4}$$

$$e^2\,t_2/\tau=4$$

$$t_2=\ln 4\tau$$

$$\frac{t_1}{t_2}=\frac{\ln\sqrt{2}}{\ln 4}=\frac{1}{2}\frac{\ln 2}{2.\ln 2}=\frac{1}{4}$$

22. A rectangular loop has a sliding connector PQ of length l and resistance $R\,\Omega$ and it is moving with a speed υ as shown. The set-up is placed in a uniform magnetic field going into the plane of the plane of the paper. The three currents I_1, I_2 and I are :



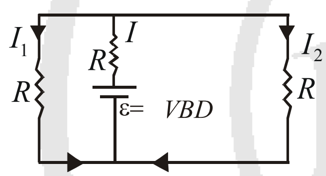
(a) $I_1 = I_2 = \frac{Bl\upsilon}{6R}, I = \frac{Bl\upsilon}{3R}$

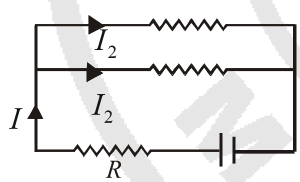
(b) $I_1 = -I_2 = \frac{Bl\upsilon}{R}, I = \frac{2Bl\upsilon}{R}$

(c) $I_1 = I_2 = \frac{Bl\upsilon}{3R}, I = \frac{2Bl\upsilon}{3R}$

(d) $I_1 = I_2 = I = \frac{Bl\upsilon}{R}$

Sol. **(c)**





$$I = \frac{2VBl}{3R}$$

$$I_1 = I_2 = \frac{I}{2} = \frac{VBl}{3R}$$

23. The equation of a wave on a string of linear mass density $0.04\,kg\,m^{-1}$ is given by

$$= 0.02(m)\sin\left[2\pi\left(\frac{t}{0.04(s)} - \frac{x}{0.50(m)}\right)\right].$$

The tension in the string is

(a) $6.25N$ (b) $4.0N$

(c) $12.5\,N$ (d) $0.5N$

Sol. **(a)**

$$\mu = \frac{m}{L} = 0.04\,kg/m$$

$$y = 0.02(m)\,Sm\left[2\pi\left(\frac{t}{0.04} - \frac{x}{0.50}\right)\right]$$

$T =$ Tension:

$$w = \frac{2\pi}{0.04}$$

$$V = \sqrt{\frac{T}{\mu}}, \qquad k = \frac{2\pi}{0.50}$$

$$V^2 = \frac{T}{\mu}$$

$$T = \mu V^2 = \mu \cdot \left(w/k\right)^2$$

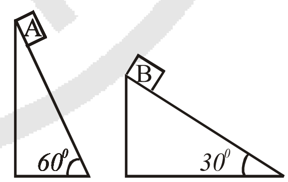
$$= \mu \cdot \left(w/k\right)^2$$

$$= 0.04 \times \left(\frac{2\pi}{0.04} \times \frac{0.50}{2\pi}\right)^2$$

$$= 0.04 \times \frac{1}{0.04} \times \frac{0.50 \times 0.50}{0.04}$$

$$= \frac{50}{4} \times 0.50 = \frac{25.0}{4} = 6.25\,N$$

24. Two fixed frictionless inclined planes making an angle 30^0 and 60^0 with the vertical are shown in the figure. Two blocks A and B are placed on the two planes. What is the relative vertical acceleration of A with respect to B?



(a) $4.9\,ms^{-2}$ in vertical direction

(b) $4.9\,ms^{-2}$ in horizontal direction

(c) $9.8\,ms^{-2}$ in vertical direction

(d) Zero

Sol.(a) Acceleration in Ist case $= \frac{3g}{4}$

Acceleration in IInd case $= \frac{g}{4}$

Difference $= 4.9\,m/s^2$

25. For a particle in uniform circular motion, the acceleration $\vec{a}$ at a point $P(R,\theta)$ on the circle of radius R is (Here θ is measured from the x-axis)

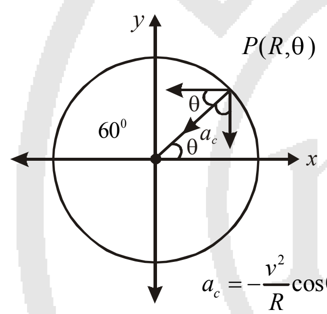
(a) $\frac{\upsilon^2}{R}\hat{i}+\frac{\upsilon^2}{R}\hat{j}$

(b) $-\frac{\upsilon^2}{R}\cos\theta\hat{i}+\frac{\upsilon^2}{R}\sin\theta\,\hat{j}$

(c) $-\frac{\upsilon^2}{R}\sin\theta\hat{i}+\frac{\upsilon^2}{R}\cos\theta\,\hat{j}$

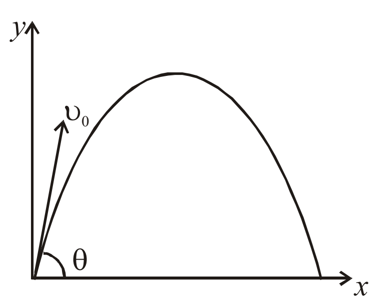
(d) $-\frac{\upsilon^2}{R}\cos\theta\,\hat{i}-\frac{\upsilon^2}{R}\sin\theta\,\hat{j}$

Sol. **(d)**



$a_c=-\frac{v^2}{R}\cos\theta\; i-\frac{v^2}{R}\sin\theta\; j$

26. A small particle of mass m is projected at an angle θ with the x-axis with an initial velocity υ_0 in the x-y plane as shown in the figure. At a time $t<\frac{\upsilon_0\sin\theta}{g}$, the angular momentum of the particle is :



(a) $\frac{1}{2}mg\upsilon_0 t^2\cos\theta\,\hat{i}$ (b) $-mg\,\upsilon_0 t^2\cos\theta\,\hat{j}$

(c) $mg\,\upsilon_0 t\cos\theta\,\hat{\kappa}$ (d) $-\frac{1}{2}mg\upsilon_0 t^2\cos\theta\,\hat{k}$

Where $\hat{i},\hat{j}$ and $\hat{k}$ are unit vectors along x, y and z -axis respectively.

Sol.(d) $\vec{L}=\vec{r}\times\vec{P}$ $\qquad\vec{r}=xi+7j$

$x=u_0\cos\theta t$

$y=u_0\sin\theta\cdot t-\frac{1}{2}gt^2$

$\vec{p}=m\vec{v}=m(v_x j+v_y j)$

$=m(u_0\cos\theta\, j+(v_0\sin\theta-gt)j$

$$\vec{L}=\begin{vmatrix} i & j & k\\ u_0\cos\theta t & u_0\sin\theta t-\frac{1}{2}gt^2 & 0\\ mu_0\cos\theta & mu_0\sin\theta-mgt & 0\end{vmatrix}$$

$=k\,(mu_0^2\cos\theta\sin\theta\cdot t=mu_0\cos gt^2)$

$-mu_0^2u_0\sin\theta t+\frac{1}{2}m\,u_0\cos\theta.gt^2$

$=-\frac{1}{2}mu_0\cos\theta gt^2\hat{k}$

27. Two identical charged spheres are suspended by strings of equal lengths. The strings make an angle of 30^0 with each other. When suspended in a liquid of density $0.8\,g\,cm^{-3}$, the angle remains the same. If density of the material of the sphere is $1.6\,g\,cm^{-3}$, the dielectric constant of the liquid is

(a) 1 (b) 4

(c) 3 (d) 2

Sol.(d) $\rho_e=0.8$

$\rho=1.6$

$k=?$

$\tan 15^0=\frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{4l^2\sin^2 15\times mg}$ ....(1)

In water :

$T\sin 15=\frac{q^2}{4\pi\varepsilon_0 k_4 l^2\sin 15^0}$ ....(2)

$T\cos 15+\beta=mg$

$T\cos 15=mg-\beta$

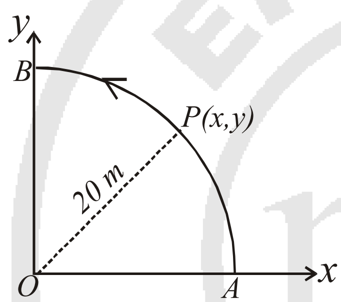
$= v \times 1.6 - v \times 0.8g \qquad = v \times 0.8g$

$T\cos 15^0 = \dfrac{mg}{2}$ .....(3)

$\tan 15^0 = \dfrac{q^2 \times 2}{4\pi\varepsilon_0 k 4l^2 \sin 15^0 mg}$ .....(4)

$\Rightarrow K = 2$

28. A point P moves in counter-clockwise direction on a circular path as shown in the figure. The movement of 'P' is such that it sweeps out a length $s = t^3 + 5$, where s is in metres and t is in seconds. The radius of the path is $20\,m$. The acceleration of 'P' when $t = 2s$ is nearly



(a) $14\,m/s^2$ (b) $13m/s^2$

(c) $12m/s^2$ (d) $7.2\,m/s^2$

Sol.(a) $S = t^3 + 5$

$\dfrac{ds}{dt} = 3t^2 = V$

$a_T = \dfrac{d^2s}{dt^2} = 6t$

$t = 2\sec$

$a_T = 6.2 = 12m/s^2$

$a_c = \dfrac{v^2}{R} = \dfrac{(3t^2)^3}{R}$

$a_c = \dfrac{9t^4}{20} = \dfrac{9 \times 2^4}{20} = \dfrac{9 \times 16}{20} \qquad = \dfrac{36}{5}$

$a = \sqrt{a_T^2 + a_c^2} \qquad = \sqrt{(12)^2 + \left(\dfrac{26}{5}\right)^2}$

$\simeq 14m/s^2$

29. The potential energy function for the the force between two atoms in a diatomic molecule is approximately given by $U(x) = \dfrac{a}{x^{12}} - \dfrac{b}{x^6}$, where a and b are constants and x is the distance between the atoms. If the dissociation energy of the molecule is $D = [U(x = \infty) - U_{\text{at equilibrium}}]$, D is :

(a) $\dfrac{b^2}{6a}$ (b) $\dfrac{b^2}{2a}$

(c) $\dfrac{b^2}{12a}$ (d) $\dfrac{b^2}{4a}$

Sol.(d) $U(x) = \dfrac{a}{x^{12}} - \dfrac{b}{x^6} \qquad U(\infty) = 0$

$\dfrac{dU}{dx} = -12\dfrac{a}{x^{13}} + 6\dfrac{b}{x^7} = 0$ for equilibrium

$\dfrac{+2a}{x^{13}} = \dfrac{b}{x^7}$

$2ax^7 = x^{13}b$

$x^{13}b = x^7\,2a$

$x^6 b = 2a$

$x^6 = \dfrac{2a}{b} \Rightarrow x^{12} = \dfrac{4a^2}{b^2}$

$U\left(x^6 = \dfrac{2a}{b}\right) = \dfrac{a}{\dfrac{4a^2}{b^2}} = \dfrac{b^2}{4a}$

$\therefore D = U(x = \infty) - U$ at equation $= \dfrac{b^2}{4a}$

30. Two conductors have the same resistance at 0^0C but their temperature coefficients of resistance are α_1 and α_2. The respective temperature coeefficients of their series and parallel combinations are nearly :

(a) $\dfrac{\alpha_1 + \alpha_2}{2}, \dfrac{\alpha_1 + \alpha_2}{2}$ (b) $\dfrac{\alpha_1 + \alpha_2}{2}, \alpha_1 + \alpha_2$

(c) $\alpha_1 + \alpha_2, \dfrac{\alpha_1 + \alpha_2}{2}$ (d) $\alpha_1 + \alpha_2, \dfrac{\alpha_1\alpha_2}{\alpha_1 + \alpha_2}$

Sol. (a)

PART - "B" CHEMISTRY

31. In aqueous solution the ionization constants for carbonic acid are

$K_1 = 4.2 \times 10^{-7}$ and $K_2 = 4.8 \times 10^{-11}$

Select the correct statements for a saturated 0.034 M solution of the carbonic acid

(a) The concentration of H^+ is double that of CO_3^{2-}

(b) The concentration of CO_3^{2-} is 0.034 M.

(c) The concentration of CO_3^{2-} is greater than that of HCO_3^-.

(d) The concentration of H^+ and HCO_3^- are approximately equal.

Sol.(d) Since $K_1 > K_2$

2nd ionisation of H_2CO_3 can be neglected in compare to first.

$\because \left(H^+\right) = \left(HCO_3^-\right)$

32. Solubility product of silver bromide is 5.0×10^{-13}. the quantity of potassium bromide (molar mass taken as $120 g\, mol^{-1}$) to be added to 1 litre of $0.05M$ solution of silver nitrate to start the precipitation of $AgBr$ is :

(a) $5.0 \times 10^{-8} g$ (b) $1.2 \times 10^{-10} g$

(c) $1.2 \times 10^{-9} g$ (d) $6.2 \times 10^{-5} g$

Sol.(c) $\left(Ag^+\right)$ in $AgBr = 0.05$

For $AgBr$ to be precipitate

$K_{ip} > K_{sp}$

$\left(Ag^+\right)\left(Br^-\right) > 5 \times 10^{-13}$

$\left(Br^-\right) > \dfrac{5 \times 10^{-13}}{0.05}$

$\left(Br^-\right) > 10^{-11}$

No. of mole of Br^- in 1 litre > 10^{-11} mol

$\therefore$ no of mole of KBr > 10^{-11} mol

wt of KBr > $10^{-11} \times 120 gm$

= 1.2×10^{-9} gm

33. The correct sequence which shows decreasing order of the ionic radii of the elements is :

(a) $O^{2-} > F^- > Na^+ > Mg^{2+} > Al^{3+}$

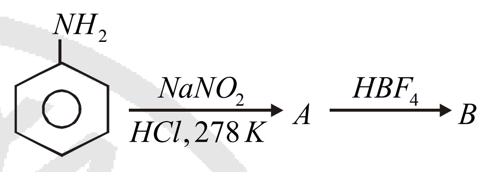
(b) $Al^{3+} > Mg^{2+} > Na^+ > F^- > O^{2-}$

(c) $Na^+ > Mg^{2+} > Al^{3+} > O^{2-} > F^-$

(d) $Na^+ > F^- > Mg^{2+} > O^{2-} Al^{3+}$

Sol.(a) Radius $\propto$ effective nuclear charge.

34. In the chemical reaction,



the compounds 'A' and 'B' respectively are-

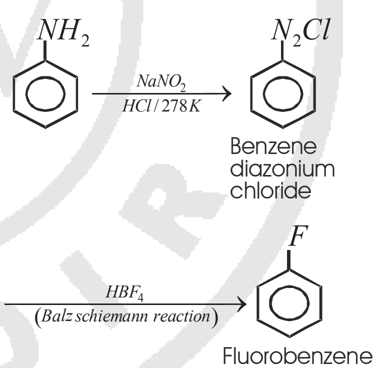
(a) nitrobenzene and chlorobenzene

(b) nitrobenzene and fluorobenzene

(c) phenol and benzene

(d) benzene diazonium chloride and fluorobenzene

Sol. **(d)**



35. If 10^{-4} dm^3 of water is introduced into a $1.0\, dm^3$ flask at 300 K, how many moles of water are in the vapour phase when equilibrium is established ?

(Given : Vapour pressure of H_2O at $300\, K$ is 3170 Pa; $R = 8.314\, J\, K^{-1}\, mol^{-1}$)

(a) $1.27 \times 10^{-3}\, mol$ (b) $5.56 \times 10^{-3}\, mol$

(c) $1.53 \times 10^{-2}\, mol$ (d) $4.46 \times 10^{-2}\, mol$

Sol.(a) $PV = nRT$

$\dfrac{3170}{1.03 \times 10^5}\left(1 - 10^{-4}\right) = n \times 0.082 \times 300$

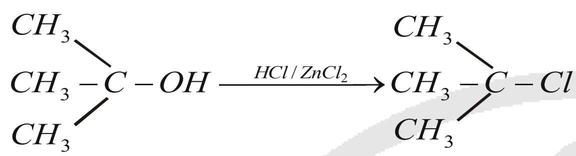
$$n = \frac{3170}{1.03 \times 10^5 \times 24.6} = 1.27 \times 10^{-3} \text{ mol.}$$

36. From amongst the following alcohols the one that would react fastest with conc. HCl and anhydrous $ZnCl_2$, is -

(a) 1-Butanol (b) 2-Butanol

(c) 2-Methylpropan-2-ol (d) 2-Methylpropanol

Sol. **(c)**



Tertiary alcohol reacts fastest

37. If sodium sulphate is considered to be completely dissociated into cations and anions in aqueous solution, the change in freezing point of water (ΔT_f), when 0.01 mol of sodium sulphate is dissolved in 1 kg of water, is $(K_f = 1.86\, K\, kg\, mol^{-1})$:

(a) $0.0186\,K$ (b) $0.0372\,K$

(c) $0.0558\,K$ (d) $0.0744\,K$

Sol.(c) $\Delta T_f = 3 \times 0.01 \times 1.86 = 0.0558K$

38. Three reactions involving $H_2PO_4^-$ are given below :

(i) $H_3PO_4 + H_2O \rightarrow H_3O^+ + H_2PO_4^-$

(ii) $H_2PO_4^- + H_2O \rightarrow HPO_4^{2-} + H_3O^+$

(ii) $H_2PO_4^- + OH^- \rightarrow H_3PO_4 + O^{2-}$

In which of the above does $H_2PO_4^-$ act as an acid ?

(a) (i) only (b) (ii) only

(c) (i) and (ii) (d) (iii) only

Sol.(b) $H_3PO_4 + H_2O \rightarrow H_3O^+ + H_2PO_4^-$ (Conjugate base)

$H_2PO_4^- + H_2O \rightarrow H_3O^+ + HPO_4^{2-}$ (Base)

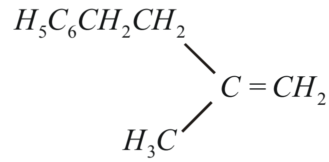
$H_2PO_4^- + OH^- \rightarrow H_3PO_4 + O_2^{2-}$ (Acid)

in only II step $H_2PO_4^-$ is acting as an acid.

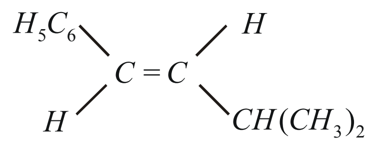
39. The main product of the following reaction is

$$C_6H_5CH_2CH(OH)CH(CH_3)_2 \xrightarrow{conc.H_2SO_4}$$

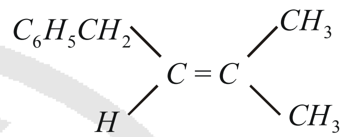
(a)



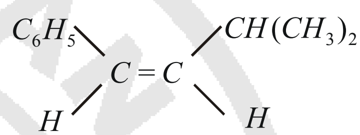
(b)



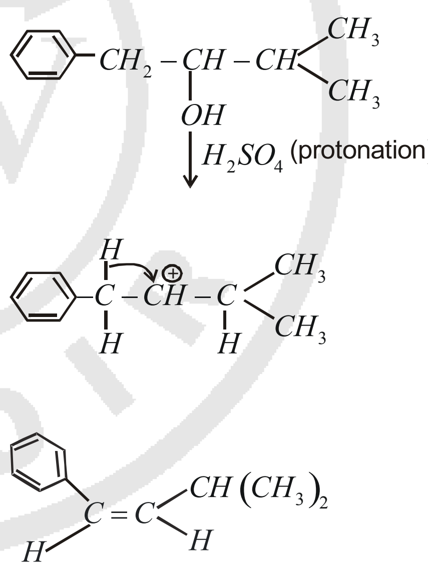
(c)



(d)



Sol.(d)



Conjugate alkene is of lower energy.

40. The energy required to break one mole of $Cl - Cl$ bonds in Cl_2 is $242\,kJ\,mol^{-1}$. The longest wavelength of light capable of breaking a single $Cl - Cl$ bond is

($c = 3 \times 10^8\, ms^{-1}$ and $N_A = 6.02 \times 10^{23}\, mol^{-1}$)

(a) 494 nm (b) 594

(c) 640 nm (d) 700 nm

Sol.(a) $\frac{242 \times 10^3}{6.023 \times 10^{23}} = \frac{hC}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{\lambda}$

$$\lambda = \frac{6.63\times10^{-34}\times3\times10^{8}\times6.023\times10^{23}}{242\times10^{3}}$$

$$= 0.494\times10^{-6}m \qquad = 494\times10^{-9}m$$

$$= 494nm$$

41. 29.5 mg of an organic compounds containing nitrogen was digested according to kjeldahl's method and the evolved ammonia was absorbed in 20 mL of $0.1M\ HCl$ solution. The excess of the acid required $15mL$ of 0.1 $M\ NaOH$ solution for complete neutralization. The percentage of nitrogen in the compounds is :

(a) 29.5 (b) 59.5
(c) 47.4 (d) 23.7

Sol.(d) % of N = $\frac{1.4\times unused\ meq.of\ HCl}{wt\ of\ O.C.}$

$$= \frac{1.4\times(20\times0.1-15\times0.1)}{29.5\times10^{-3}} = 23.72\%$$

42. Ionisation energy of He^{+} is $19.6\times10^{-18}\ J\ atom^{-1}$. The energy of the first stationary state $(n=1)$ of Li^{2+} is :

(a) $8.82\times10^{-17}\ J\ atom^{-1}$

(b) $4.41\times10^{-16}\ J\ atom^{-1}$

(c) $-4.41\times10^{-17}\ J\ atom^{-1}$

(d) $-2.2\times10^{-15}\ J\ atom^{-1}$

Sol.(c) E_1 of He^{+} = E_1 of $H-$atom $\times\frac{Z^2}{n^2}$

$= E_1$ of H-atom $\times 2^2$

$\therefore\ E_1$ of H-atom $= \frac{E_1\ of\ He^{+}}{2^2}$

$$= -\frac{ionisation\ energy\ of\ He^{+}}{4}$$

$$= -\frac{19.6\times10^{-18}}{4}$$

E_1 of Li^{+2} = E_1 of H-atom $\times\frac{3^2}{1^2}$

$$= \frac{-19.6\times10^{-18}}{4}\times9 \qquad = -4.41\times10^{-17}J$$

43. On mixing, heptane and octane form an ideal solution. At 373 K, the vapour pressures of the two liquid components (heptane and octane) are 105 kPa and 45 kPa respectively. Vapour pressure of the solution obtained by mixing $25.0\,g$ of heptane and $35g$ of octane will be (molar mass of heptane $=100\,g\,mol^{-1}$ and of octane $=114g\,mol^{-1}$)

(a) $144.5\,kPa$ (b) $72.0\,kPa$

(c) $26.1\,kPa$ (d) $96.2\,kPa$

Sol.(b) $P_{Hept} = 105KPa$ $\quad 25g\ Hept$

$P_{Oct} = 45Kpa$ $\quad 35g\ Oct$

M.w 100g/mol of heptane
M.w 114g/mol of octane

$$P_T = P_A^0 X_A + P_B^0 X_B$$

$$= 105.\frac{0.25}{0.25+0.3} + 45.\frac{0.3}{0.25+0.3}$$

$$= 105.\frac{0.25}{0.55} + 45.\frac{0.3}{0.55}$$

$$= 105.(0.45) + 45.(0.545)$$

$$= 47.25 + 24.525$$

$$= 71.775 \approx 72$$

44. Which one of the following has an optical isomer ?

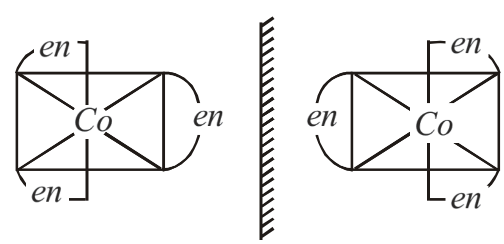
(a) $[Zn(en)_2]^{2+}$

(b) $[Zn(en)(NH_3)_2]^{2+}$

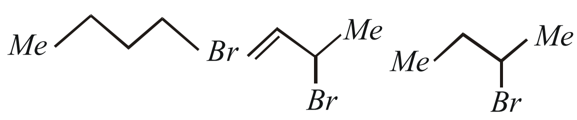
(c) $[Co(en)_3]^{3+}$

(d) $[Co(H_2O)_4(en)]^{3+}$

Sol.(c) $[M(aa)_3]^{n+}$ type will show optical isomerism



45. Consider the following bromides :



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The correct order of S_N1 reactivity is :

(a) A > B > C (b) B > C > A

(c) B > A > C (d) C > B > A

Sol.(b) $B > C > A$ more is stablity of carbocation favour SN[1] mechanism.

46. One mole of a symmetrical alkene on ozonolysis gives two moles of an aldehyde having a molecular mass of $44\,u$. The alkene is :

(a) ethene (b) propene

(c) 1-butene (d) 2- butene

Sol. **(d)**

O|O

$CH_3-CH=CH-CH_3$

$2CH_3CHO$

molecular weight of aldehyde = 44

47. Consider the reaction :

$$Cl_2(aq)+H_2S(aq)\rightarrow S(s)+2H^+(aq)+2Cl^-(aq)$$

The rate equation for this reaction is

rate $=k[Cl_2][H_2S]$

Which of these mechanisms is/are consistent with this rate equation ?

A . $Cl_2+H_2S\rightarrow H^++Cl^-+Cl^++HS^-$ (slow)

$Cl^++HS^-\rightarrow H^++Cl^-+S$ (fast)

B . $H_2S\Leftrightarrow H^++HS^-$ (fast equilibrium)

$Cl_2+HS^-\rightarrow 2Cl^-+H^++S$ (slow)

(a) A only (b) B only

(c) Both A and B (d) Neither A nor B

Sol.(a) (i) $Cl_2+H_2S\rightarrow H^++Cl^-+Cl^++HS$

$R=K\left[Cl_2\right]\left[H_2S\right]$

Hence it is consistent

(ii) $R=K^1\left[Cl_2\right]\left[HS\right]$

$$K_{eq.}=\frac{\left[H^+\right]\left[HS^-\right]}{\left[H_2S\right]}$$

$$\therefore \left[HS\right]=\frac{K_{eq.}.\left(H_2S\right)}{\left(H^+\right)}$$

$$\therefore R=K^1K_{eq.}\left(Cl_2\right).\left(\frac{H_2S}{H^+}\right)$$

Which is not consistent

48. The Gibbs energy for the decomposition of Al_3O_3 at 500^0C is as follows :

$$\frac{2}{3}Al_2O_3\rightarrow\frac{4}{3}Al+O_2,\Delta_rG=+966\,J\,mol^{-1}$$

The potential difference needed for electrolytic reduction of Al_2O_3 at 500^0C is at least :

(a) $5.0V$ (b) $4.5V$

(c) $3.0V$ (d) $2.5V$

Sol.(d) $\frac{2}{3}Al_2O_3\rightarrow\frac{4}{3}Al+O_2$

$\Delta G=+966KJ$

$\Delta G=-nF_{Cell}$

$966\times10^3=-4\times96500\times-E_{Cell}$

$E_{Cell}=-\dfrac{966\times10^3}{4\times96500}$ = 2.5V

49. The correct order of increasing basicity of the given conjugate bases $(R=CH_3)$ is :

(a) $RCO\bar{O}<HC\equiv\bar{C}<\bar{N}H_2<\bar{R}$

(b) $RCO\bar{O}<HC\equiv\bar{C}<\bar{R}<\bar{N}H_2$

(c) $\bar{R}<HC\equiv\bar{C}<RCO\bar{O}<\bar{N}H_2$

(d) $RCO\bar{O}<\bar{N}H_2<HC\equiv\bar{C}<\bar{R}$

Sol.(a) Acidic nature

$R-COOH>H-C\equiv CH>NH_3>RH$

This is order of basicity of conjugate base

$RCOO^-<H-C\equiv C^-<NH_2^-<R^-$

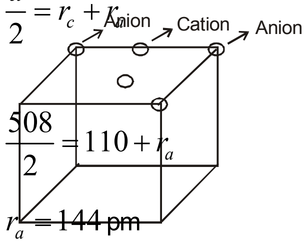
50. The edge length of a face centered cubic cell of an ionic substance is 508 pm. If the radius of the cation is 110 pm, the radius of the anion is:

(a) 144 pm (b) 288 pm

(c) 398 pm (d) 618 pm

Sol. **(a)**

$\frac{a}{2} = r_c + r_a$



$\frac{508}{2} = 110 + r_a$

$r_a = 144$ pm

51. Out of the following, the alkene that exhibits optical isomerism is :

(a) 2-methyl-2-pentene

(b) 3-methyl-2-pentene

(c) 4-methyl-1-pentene

(d) 3-methyl-1-pentene

Sol.(d) $\overset{1}{H_2C} = \overset{2}{CH} - \overset{H}{\underset{CH_3}{\overset{3|}{C}}} - \overset{4}{CH_2} - \overset{5}{CH_3}$

All the groups at carbon 3 is diffrent hence it is chiral carbon it will exhibit optical isomerism.

52. For a particular reversible reaction at temperature $T, \Delta H$ and ΔS were found to be both $+ve$. If T_e is the temperature at equilibrium, the reaction would be spontaneous when :

(a) $T = T_e$ (b) $T_e > T$

(c) $T > T_e$ (d) T_e is 5 times T

Sol.(c) For reaction to be spontaneous $T > \frac{\Delta H}{\Delta S}$

and at equilibrium $T_e = \frac{\Delta H}{\Delta S}$

$\therefore T > T_e$

53. Percentages of free space in cubic close packed structure an in body centered packed structure are respectively :

(a) 48 % and 26 % (b) 30 % and 26 %

(c) 26 % and 32 % (d) 32 % and 48 %

Sol. **(c)**

54. The polymer containing strong intermolecular forces e.g. hydrogen bonding, is :

(a) natural rubber (b) teflon

(c) nylon 6, 6 (d) polystyrene

Sol. **(c)**

Nylon 6, 6 called fiber which has strong intermolecular forces such as H-bonding.

55. At $25^0 C$, the solubility product of $Mg(OH)_2$ is 1.0×10^{-11}. At which pH, will Mg^{2+} ions start precipitating in the form of $Mg(OH)_2$ from a solution of 0.001 $M\ Mg^{2+}$ ions ?

(a) 8 (b) 9

(c) 10 (d) 11

Sol.(c) $\left(Mg^{2+}\right)\left(OH^-\right)^2 > 1 \times 10^{-11}$

$\left(OH^-\right)^2 > \frac{10^{-11}}{0.001}$

$\left(OH^-\right)^2 > 10^{-8}$

$\left(OH^-\right) > 10^{-4}$

$pOH < 4$ $\therefore pH > 10$

56. The correct order of $E^0_{M^{2+}/M}$ values with negative sign for the four successive elements Cr, Mn, Fe and Co

(a) $Cr > Mn > Fe > Co$

(b) $Mn > Cr > Fe > Co$

(c) $Fe > Mn > Cr > Co$

(d) $Cr > Fe > Mn > Co$

Sol.(b) $Mn > Cr > Fe > CO$

$-1.18 > -0.91 > -0.44 > -0.27$

This is due to irregular variation of ionisation energies and the sublimation energies of atoms and hydration energy of divalent ion.

57. Biuret test is not given by :

(a) proteins (b) carbohydrates

(c) polypeptides (d) urea

Sol.(b) Biuret test gives compound containing $-\overset{O}{\overset{||}{C}} - NH -$ linkage . Since carbohydrate donot conatin $-\overset{O}{\overset{||}{C}} - NH -$ linkage hence donot give biuret test.

58. The time for half life period of a certain reaction $A \rightarrow$ Products is 1 hour. When the initial concentration of the reactant 'A' is 2.0 $mol\,L^{-1}$, how much time does it take for its concentration to come from 0.50 to 0.25

$mol\, L^{-1}$ if it is a zero order reaction ?

(a) 1 h (b) 4h

(c) 0.5 h (d) 0.25 h

Sol.(d) For zero order reaction

$$t_{1/2} = \frac{a}{2K} \qquad \therefore t_{1/2} \propto a$$

$$\frac{(t_{1/2})_1}{(t_{1/2})_2} = \frac{a_2}{a_1}$$

$$\frac{1}{(t_{1/2})_2} = \frac{2}{0.5}$$

$$(t_{1/2})_2 = \frac{1}{4} = 0.25 \text{ hr.}$$

59. A solution containing 2.675 g of $CoCl_3.6NH_3$ (molar mass $= 267.5 g\, mol^{-1}$) is passed through a cation exchanger. The chloride ions obtained in solution were treated with excess of $AgNO_3$ to give $4.78 g$ of $AgCl$ (molar mass = 143.5 $g\, mol^{-1}$). The formula of the complex is :
(At mass of Ag = 108 u)

(a) $[CoCl(NH_3)_5]Cl_2$

(b) $[Co(NH_3)_6]Cl_3$

(c) $[CoCl_2(NH_3)_4]Cl$

(d) $[CoCl_3(NH_3)_3]$

Sol.(b) $CoCl_3.6NH_3 + AgNO_3 \rightarrow AgCl$

$$n = \frac{2.675}{267.5} \qquad = \frac{4.78}{143.5}$$

$$= 0.01 \qquad = 0.033$$

Since 0.01 mole $CoCl_3.6NH_3$ gives 0.033 mole $AgCl$.

Hence 1 mole $CoCl_3.6NH_3$

gives $= \frac{0.033}{0.01}$ = 3.3 mole

= 3 mole

Hence no. of chloride ion given by 1 mole compound = 3

$\therefore$ Ans = $[Co(NH_3)_6]Cl_3$

60. The standard enthalpy of formation of NH_3 is $-46.0\, kJ\, mol^{-1}$. If the enthalpy of formation of H_2 from its atoms is $-436\, kJ\, mol^{-1}$ and that of N_2 is $-712\, kJ\, mol^{-1}$, the average bond enthalpy of $N-H$ bond in NH_3 is :

(a) $-1102\, kJ\, mol^{-1}$ (b) $-964\ kJ\, mol^{-1}$

(c) $+352\ kJ\, mol^{-1}$ (d) $+1056\ kJ\, mol^{-1}$

Sol.(c) $\frac{1}{2}N_2 + \frac{3}{2}H_2 \rightarrow NH_3$, $\Delta H = -46 kJ$

$$\Delta H = -46 = \left[\frac{1}{2} B.E. of (N \equiv N) + \frac{3}{2} B.E. of (H-H)\right]$$

$$-\left[3 \times B.E. of (N-H)\right]$$

$$-46 = \left[\frac{1}{2} \times 712 + \frac{3}{2} \times 436\right] - 3[N-H]$$

$$= (356 + 654) - 3(N-H)$$

$$3(N-H) = 356 + 654 + 46 = 1056$$

$$(N-H) = \frac{1056}{3} = 352 \text{ KJ}$$

Part C - MATHEMATICS

61 Let $\cos(\alpha+\beta)=\frac{4}{5}$ and let $\sin(\alpha-\beta)=\frac{5}{13}$, where $0\le\alpha,\beta\le\frac{\pi}{4}$.

Then $\tan 2\alpha =$

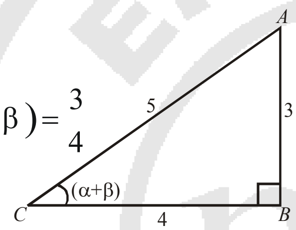
(a) $\frac{56}{33}$ (b) $\frac{19}{12}$

(c) $\frac{20}{7}$ (d) $\frac{25}{16}$

Sol.(a) $\cos(\alpha+\beta)=\frac{4}{5}$ $\quad\sin(\alpha-\beta)=\frac{5}{13}$

$\tan 2\alpha=\tan\left[(\alpha+\beta)+(\alpha-\beta)\right]$

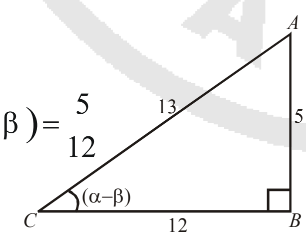
$\tan(\alpha+\beta)=\frac{3}{4}$



$$=\frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\cdot\tan(\alpha-\beta)}$$

$$=\frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\cdot\frac{5}{12}}=\frac{\frac{36+20}{48}}{\frac{48-15}{48}}=\frac{56}{33}$$

$\tan(\alpha-\beta)=\frac{5}{12}$



62. Let S be a non-empty subset of R.
Consider the following statement :
P : There is a rational number $x\in S$ such that $x>0$.
Which of the following statements is the negation of the statement P ?

(a) There is no rational number $x\in S$ such that $x\le 0$.

(b) Every rational number $x\in S$ satisfies $x\le 0$.

(c) $x\in S$ and $x\le 0\Rightarrow x$ is not rational

(d) There is a rational number $x\in S$ such that $x\le 0$.

Sol.(b) Every rational number $x\in S$ satisfies $x\le 0$

63. Let $\vec{a}=\hat{j}-\hat{k}$ and $\vec{c}=\hat{i}-\hat{j}-\hat{k}$. Then the vector $\vec{b}$ satisfying $\vec{a}\times\vec{b}+\vec{c}=\vec{0}$ and $\vec{a}\cdot\vec{b}=3$ is

(a) $2\hat{i}-\hat{j}+2\hat{k}$ (b) $\hat{i}-\hat{j}-2\hat{k}$

(c) $\hat{i}+\hat{j}-2\hat{k}$ (d) $-\hat{i}+\hat{j}-2\hat{k}$

Sol.(d) $\vec{a}=\hat{j}-\hat{k}$ and $\vec{c}=\hat{i}-\hat{j}-\hat{k}$

Let $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$

$\because \vec{a}\times\vec{b}+\vec{c}=0$, $\vec{a}\times\vec{b}=-\vec{c}$

$$\begin{vmatrix} i & j & k \\ 0 & 1 & -1 \\ b_1 & b_2 & b_3\end{vmatrix}=-\hat{i}+\hat{j}+\hat{k}$$

$i(b_3+b_2)-\hat{j}(b_1)+k(-b_1)=-\hat{i}+\hat{j}+\hat{k}$

$b_3+b_2=-1$ .....(a)

$b_1=-1$ .....(b)

$\vec{a}\cdot\vec{b}=3$

$(j-k)\cdot(b_1i+b_2j+b_3k)=3$

$b_2-b_3=3$ ....(c)

Solve (a) and (c)

$2b_2=2$ $\quad b_2=1$ $\quad b_3=-2$

$\therefore b_1=-1$ $\quad b_2=1$ $\quad b_3=-2$

Hence $\vec{b}=-\hat{i}+\hat{j}-2\hat{k}$

64. The equation of the tangent to the curve $y=x+\frac{4}{x^2}$, that is parallel to the x-axis, is

(a) $y=1$ (b) $y=2$

(c) $y=3$ (d) $y=0$

Sol.(c) Parallel to x-axis i.e. $\frac{dy}{dx}=0$

$\frac{dy}{dx}=1-8x^{-3}=0$ $\quad\Rightarrow \boxed{x=2}$

put $x=2$ in equation of curve, we get

$y = 3$

Hence equation of tangent is

$y - 3 = 0(x-2) \qquad \Rightarrow y = 3$

65. Solution of the differential equation

$\cos x\, dy = y(\sin x - y)dx, 0 < x < \dfrac{\pi}{2}$ is

(a) $y \sec x = \tan x + c$

(b) $y \tan x = \sec x + c$

(c) $\tan x = (\sec x + c)y$

(d) $\sec x = (\tan x + c)y$

Sol.(d) $\dfrac{dy}{dx} = \dfrac{y(\sin x - y)}{\cos x}$

$\dfrac{dy}{dx} = y\tan x - y^2 \sec x$

$\Rightarrow -\dfrac{1}{y^2}\dfrac{dy}{dx} + \tan x \dfrac{1}{y} = \sec x$

Let $\dfrac{1}{y} = v \Rightarrow -\dfrac{1}{y^2}\dfrac{dy}{dx} = \dfrac{dv}{dx}$

$\Rightarrow \dfrac{dv}{dx} + (\tan x)v = \sec x$

$I.F. = e^{\int \tan x} = e^{\log \sec x} = \sec x$

$\Rightarrow v(\sec x) = \int \sec x \sec x\, dx + c$

$\Rightarrow \dfrac{1}{y}\sec x = \tan x + c$

$\Rightarrow \sec x = (\tan x + c)y$

66. The area bounded by the curves $y = \cos x$ and $y = \sin x$ between the ordinates $x = 0$ and $x = \dfrac{3\pi}{2}$ is

(a) $4\sqrt{2}+2$ (b) $4\sqrt{2}-1$

(c) $4\sqrt{2}+1$ (d) $4\sqrt{2}-2$

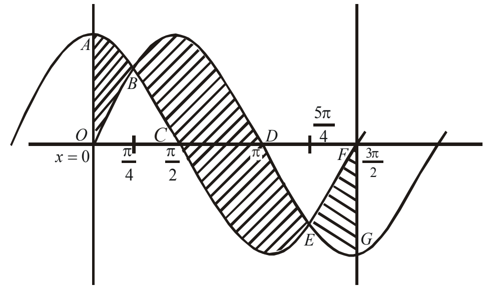
Sol.(d) Area of curve

$OAB = \int_0^{\pi/4}(\cos x - \sin x)dx$

$= [\sin x + \cos x]_0^{\pi/4}$

$= \dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{2}} - (0+1) = \sqrt{2} - 1$

Similarly, area of curve $EFG = \sqrt{2} - 1$



Now, area of curve

$BCDE = \int_{\pi/4}^{5\pi/4}(\sin x - \cos x)dx$

$= (-\cos x - \sin x)_{x=\frac{\pi}{4}}^{x=\frac{5\pi}{4}}$

$= -\left[-\dfrac{1}{\sqrt{2}} - \dfrac{1}{\sqrt{2}} - \dfrac{1}{\sqrt{2}} - \dfrac{1}{\sqrt{2}}\right]$

$= \dfrac{4}{\sqrt{2}} = 2\sqrt{2}$

$\Rightarrow$ Net area

$= (\sqrt{2}-1) + (\sqrt{2}-1) + 2\sqrt{2} = 4\sqrt{2} - 2$

67 If two Tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles, then the locus of P is

(a) $2x+1=0$ (b) $x=-1$

(c) $2x-1=0$ (d) $x=1$

Sol.(d) If tangents of a parabola are $\perp^r$, then it meets on the directrix

$\therefore$ equation of directrix $\Rightarrow x = -1$

$\boxed{x=-1}$

68. If the vectors

$\vec{a} = \hat{i} - \hat{j} + 2\hat{k}, \vec{b} = 2\hat{i} + 4\hat{j} + \hat{k}$ and

$\vec{c} = \lambda\hat{i} + \hat{j} + \mu\hat{k}$ are mutually orthogonal, then $(\lambda, \mu) =$

(a) $(2,-3)$ (b) $(-2,3)$

(c) $(3,-2)$ (d) $(-3,2)$

Sol.(d) $\vec{a}\cdot\vec{b} = 0$

$\Rightarrow (i - j + 2k)\cdot(2i + 4j + k) = 0$

$\vec{a}\cdot c = 0$

$\Rightarrow (i - j + 2\hat{k})\cdot(\lambda i + 4j + \mu k) = 0$

$\Rightarrow \lambda - 1 + \mu \cdot 2 = 0 \Rightarrow \lambda - 2\mu = 1$ .....(a)

$\vec{b} \cdot \vec{c} = 0$

$(2i - 4j + k) \cdot (\lambda i + \hat{j} + \mu k) = 0$

$2 \cdot \lambda + 4 + \mu = 0$

$2\lambda + \mu = -4$ .....(b)

Solve (a) and (b), we get $\left.\begin{matrix} \lambda = -3 \\ \mu = 2 \end{matrix}\right\}$

69. Consider the following relations :

$R = \{(x, y) \mid x, y$ are real number and $x = wy$ for some rational number $w\}$;

$S = \left\{\left(\frac{m}{n}, \frac{p}{q}\right)\right\} \mid m, n, p$ and q are integers such that $n, q \neq 0$ and $qm = pn\}$. Then

(a) Neither R nor S is an equivalence relation.

(b) S is an equivalence relation but R is not an equivalence relation.

(c) R and S both are equivalence relations.

(d) R is an equivalence relation but S is not an equivalence relation.

Sol.(c) For relation R,

$xRx \Rightarrow x = \omega x \Rightarrow w = 1$ is a rational number i.e. R is reflexive

Again $xRy \Rightarrow x = wy \Rightarrow w$ is a rational number $yRx \Rightarrow y = \frac{1}{w}x \Rightarrow \frac{1}{w}$ is a rational number

So, $xRy \Rightarrow yRx$ i.e. R is symmetric

Also, $xRy \Rightarrow x = wy$

$\Rightarrow w$ is a rational number .....(a)

$yRz \Rightarrow y = wz$

$\Rightarrow w$ is a rational number .....(b)

By (a) and (b) $\Rightarrow x = w^2 z \Rightarrow w^2$ is a rational number

So, $xRy,\ yRz \Rightarrow xRz$ Hence R is equivalence relation.

For relation S, $S = \left\{\left(\frac{m}{n}, \frac{p}{q}\right)\right.$ m, n, p and q are integers such that $n, q \neq 0$ and $\left.\frac{m}{n} = \frac{p}{q}\right\}$

Similarly it is an equivalance relation.

70. Let $f : R \to R$ be difined by $f(x) = \begin{cases} k - 2x, & if \quad x \le -1 \\ 2x + 3, & if \quad x > -1 \end{cases}$. If f has a local minimum at $x = -1$, then a possible value of k is

(a) 0 (b) $-\frac{1}{2}$

(c) -1 (d) 1

Sol.(c) f is minimum at $x = -1$

Hence $f(-1) \le \lim\limits_{x \to -1^+} f(x)$

$\Rightarrow K + 2 \le 1 \quad \Rightarrow \boxed{K \le -1}$

71. The number of 3×3 non-singular matrices, with four entries as 1 and all other entries as 0, is

(a) 5 (b) 6

(c) at least 7 (d) less than 4

Sol.(c)

$$\begin{bmatrix}1&1&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}1&0&1\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}1&0&0\\0&1&1\\0&0&1\end{bmatrix}\begin{bmatrix}1&0&0\\1&1&0\\0&0&1\end{bmatrix}$$

$$\begin{bmatrix}1&0&0\\0&1&0\\1&0&1\end{bmatrix}\begin{bmatrix}1&0&0\\0&1&0\\0&1&1\end{bmatrix}\begin{bmatrix}1&0&0\\0&1&1\\0&1&0\end{bmatrix} \text{ etc.}$$

are non-singular matrices.

So, at least 7 are non singular matrices.

Directions :

Questions number72 to 76 are Assertion-Reason type questions. Each of these questions contains two statements.

Statement 1 : (Assertion) and

Statement 2 : (Reason) .

Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select the correct choice.

72. Four number are chosen at random (without replacement) from the set $\{1, 2, 3, ...20\}$.

Statement 1 :
The probability that the chosen numbers when arranged in some order will form an AP is $\frac{1}{85}$.

Statement 2 :
If the four chosen numbers form an AP, then the set of all possible values of common difference is $\{\pm 1, \pm 2, \pm 3, \pm 4, \pm 5\}$.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** a correct explanation for Statment -1.

(b) Statement-1 is true, Statement-2 is false.

(c) Statement-1 is false, Statement-2 is true.

(d) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statment -1.

Sol.(b) $d = 1$, least possible A.P. 17,18,19,20
i.e. no. of A.P.'s =17

$d = 2$, least possible A.P. 14,16,18,20
i.e. no. of A.P.'s =14

$d = 3$, least possible A.P. 11,14,17,20
i.e. no. of A.P.'s =11

$d = 4$, least possible A.P. 8,12,16,20
i.e. no. of A.P.'s =8

$d = 5$, least possible A.P. 5,10,15,20
i.e. no. of A.P.'s =5

$d = 6$, least possible A.P. 2,8,14,20
i.e. no. of A.P.'s =2

$d = 7$ Not possible

$\Rightarrow n(A) = 57$

Now,

$$n(s) = {}^{20}C_4 = \frac{20.19.18.17}{4.3.2.1} = 19(17)(15)$$

Probability $= \frac{57}{19(17)(15)} = \frac{1}{85}$

Statement (a) is correct

Now $d = \{1,2,3,4,5,6\}$ or

$\{-1,-2,-3,-4,-5,-6\} \Rightarrow$ (b) is wrong

73. **Statement 1 :**
The point $A(3,1,6)$ is the mirror image of the point $B(1,3,4)$ in the plane $x - y + z = 5$.

Statement 2 :
The plane $x - y + z = 5$ bisects the line segment joining $A(3,1,6)$ and $B(1,3,4)$.

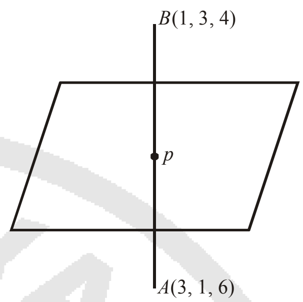
(a) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** a correct explanation for Statment -1.

(b) Statement-1 is true, Statement-2 is false.

(c) Statement-1 is false, Statement-2 is true.

(d) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statment -1.

Sol.(a) Equation of line passing through A and B



$$\Rightarrow \frac{x-1}{3-1} = \frac{y-3}{1-3} = \frac{z-4}{6-4}$$

$$\Rightarrow \frac{x-1}{2} = \frac{y-3}{-2} = \frac{z-4}{2} = k$$

d.R's $AB \Rightarrow (2, -2, 2) = (1, -1, 1)$

d.R's of plane $x - y + z = 5 \Rightarrow 1, -1, 1$

$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ $\quad \frac{2}{1} = \frac{-2}{-1} = \frac{2}{1}$

$\therefore$ Plane and line are $\perp^r$

any point on line $(2k+1, -2k+3, 2k+4)$
it lies on the plane,

$2k + 1 - (-2k + 3) + 2k + 4 = 5$

$\Rightarrow 6k + 2 = 5 \Rightarrow k = \frac{1}{2} \quad \therefore p(2, 2, 5)$

Middle point of

$$AB \Rightarrow p\left(\frac{3+1}{2}, \frac{1+3}{2}, \frac{6+4}{2}\right)$$

$\Rightarrow p(2, 2, 5)$

p, lie on the plane $2 - 2 + 5 = 5$

$\therefore$ Plane bisect the line segement AB

74. Let $S_1 = \sum_{j=1}^{10} j(j-1)\,{}^{10}C_j$, $S_2 = \sum_{j=1}^{10} j\,{}^{10}C_j$

and $S_3 = \sum_{j=1}^{10} j^2\,{}^{10}C_j$

Statement 1 :

$S_3 = 55 \times 2^9$

Statement 2 :

$S_1 = 90 \times 2^8$ and $S_2 = 10 \times 2^8$

(a) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** a correct explanation for Statment -1.
(b) Statement-1 is true, Statement-2 is false.
(c) Statement-1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statment -1.

Sol.(b) $(1+x)^{10} = \sum_{j=1}^{10} {}^{10}C_j x^j$

On diff. $10(1+x)^9 = \sum_{j=1}^{10} j \ {}^{10}C_j \ x^{j-1}$

for $x = 1$ $10.2^9 = \sum_{j=1}^{10} j^{10}C_j = S_2$ .....(a)

$\Rightarrow S_2 = 10.2^9$ which is wrong.

$(1+x)^{10} = \sum_{j=1}^{10} {}^{10}C_j \ x^j$

On diff. $10(1+x)^9 = \sum_{j=1}^{10} j \ {}^{10}C_j \ x^{j-1}$

Again diff.

$10.9(1+x)^8 = \sum_{j=1}^{10} j(j-1) \ {}^{10}C_j \ x^{j-2}$

for $x = 1$

$90.2^8 = \sum_{j=1}^{10} j(j-1) \ {}^{10}C_j = \sum_{j=1}^{10} (j^2 - j) \ {}^{10}C_j$

$\Rightarrow S_1 = 90.2^8$

Now (a)+(b) $\Rightarrow 10.2^9 + 90.2^8 = \sum_{j=1}^{10} j^2 \ {}^{10}C_j$

$\Rightarrow 10.2^8 = \sum_{j=1}^{10} j^2 \ {}^{10}C_j$

$\Rightarrow 55.2^9 = \sum_{j=1}^{10} j^2 \ {}^{10}C_j \quad \Rightarrow S-1$ is true

75. Let A be a 2×2 matrix with non-zero entries and let $A^2 = I$, where I is 2×2 identity matrix. Define $Tr(A) =$ sum of diagonal elements of A and $|A| =$ determinant of matrix A

Statement 1 :

$Tr(A) = 0$

Statement 2 :

$|A| = 1$

(a) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** a correct explanation for Statment -1.
(b) Statement-1 is true, Statement-2 is false.

(c) Statement-1 is false, Statement-2 is true.

(d) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statment -1.

Sol.(c) $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$a^2 + bc = 1, \ bc + d^2 = 1$

$\Rightarrow a^2 = d^2 \qquad \Rightarrow a = \pm d$

So, $Tr(A) \neq 0$ for $a = d$

Statement (a) is wrong

$A^2 = I \Rightarrow |A^2| = |I| \Rightarrow |A|^2 = 1 \Rightarrow |A| = 1$

Statement (b) is true

76. Let $f : R \to R$ be a continuous function defined by $f(x) = \dfrac{1}{e^x + 2e^{-x}}$.

Statement 1 :

$f(c) = \dfrac{1}{3}$, for some $c \in R$.

Statement 2 :

$0 < f(x) \leq \dfrac{1}{2\sqrt{2}}$ for all $x \in R$.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** a correct explanation for Statment -1.
(b) Statement-1 is true, Statement-2 is false.
(c) Statement-1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statment -1.

Sol.(d) $f(x) = \dfrac{e^x}{2 + e^{2x}}$ and $f'(x) = 0$

we get $e^{2x} = 2 \quad 2x = \log_e 2$

$\Rightarrow x = \log_e \sqrt{2}$

$f(x) > 0$ it is a exponential funtion

Hence $0 < f(x) < \frac{1}{2\sqrt{2}}$

Also $f(0) = \frac{1}{3}$ At $x = 0$

which is true.

77. For a regular polygon, let r and R be the radii of the inscribed and the circumscribed circles. A False statement among the following is -

(a) There is a regular polygon with $\frac{r}{R} = \frac{1}{\sqrt{2}}$

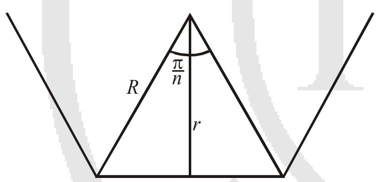
(b) There is a regular polygon with $\frac{r}{R} = \frac{2}{3}$

(c) There is a regular polygon with $\frac{r}{R} = \frac{\sqrt{3}}{2}$

(d) There is a regular polygon with $\frac{r}{R} = \frac{1}{2}$

Sol.(b) From figure, for a regular polygon

$\Rightarrow \frac{r}{R} = \cos\frac{\pi}{n}$



for $n = 3$ $\cos\frac{\pi}{3} = \frac{1}{2}$

for $n = 4$ $\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

for $n = 6$ $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

So, there is a regular polygon with $\frac{r}{R} = \frac{2}{3}$ is false.

78. If α and β are the roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{2009} + \beta^{2009} =$

(a) -1 (b) 1

(c) 2 (d) -2

Sol.(b) $x^2 - x + 1 = 0$

$\Rightarrow x = \frac{1+\sqrt{3}i}{2} = -\omega^2, x = \frac{1-\sqrt{3}i}{2} = -\omega$

So, $\alpha^{2009} + \beta^{2009} = (-\omega^2)^{2009} + (-\omega)^{2009}$

$= -\omega^{2(2009)} - \omega^{2009}$

$= -\left(\omega^{3(1339)+1}\right) - \left(\omega^{3(669)+2}\right)$

$= -\omega - \omega^2$

$= -(\omega + \omega^2) = -(-1) = 1$

79. The number of complex numbers z such that $|z-1| = |z+1| = |z-i|$ equals

(a) 1 (b) 2

(c) ∞ (d) 0

Sol.(a) Let $z = x + iy$ Now $|z-1| = |z+1|$

$\Rightarrow |(x-1)+iy| = |(x+1)+iy|$

$\Rightarrow \sqrt{(x-1)^2 + y^2} = \sqrt{(x+1)^2 + y^2}$

$\Rightarrow x^2 - 2x + 1 + y^2 = x^2 + 2x + 1 + y^2$

$\Rightarrow 4x = 0 \quad \Rightarrow x = 0$ .....(a)

Again $|z+1| = |z-i|$

$\Rightarrow |(x+1)+iy| = |x+(y-1)i|$

$\Rightarrow \sqrt{(x+1)^2 + y^2} = \sqrt{x^2 + (y-1)^2}$

$\Rightarrow x^2 + y^2 + 2x + 1 = x^2 + y^2 - 2y + 1$

$\Rightarrow x = -y$ ......(b)

Also $|z-1| = |z-i|$

$\Rightarrow |(x-1)+iy| = |x+(y-1)i|$

$\Rightarrow \sqrt{(x-1)^2 + y^2} = \sqrt{x^2 + (y-1)^2}$

$= \sqrt{x^2 + y^2 - 2x + 1} = \sqrt{x^2 + y^2 - 2y + 1}$

$\Rightarrow x = y$ ......(c)

BY (a), (b), (c)

$\Rightarrow x = 0, y = 0 \Rightarrow x + iy = 0 + i0$

Only one complex number

80. A line AB in three-dimensional space makes angles $45°$ and $120°$ with the positive x-axis and the positive y-axis respectively. If AB makes an acute angle θ with the positive z-axis, then θ equals

(a) $45°$ (b) $60°$

(c) $75°$ (d) $30°$

Sol.(b) $\alpha = 45°, \beta = 120°, \gamma = \theta$

$\cos^2\alpha + \cos^2\beta + \cos^2\theta = 1$

$\Rightarrow \left(\frac{1}{2}\right) + \left(-\frac{1}{2}\right)^2 + \cos^2\theta = 1$

$\frac{1}{2} + \frac{1}{4} + \cos^2\theta = 1 \qquad \Rightarrow \frac{3}{4} + \cos^2\theta = 1$

$\Rightarrow \cos^2\theta = \frac{1}{4} \qquad \Rightarrow \cos\theta = \pm\frac{1}{2}$

$\theta = 60°$ (acute angle).

81. The line L given by $\frac{x}{5} + \frac{y}{b} = 1$ passes through the point $(13, 32)$. The line K is parallel to L and has the equation $\frac{x}{c} + \frac{y}{3} = 1$. Then the distance between L and K is

(a) $\sqrt{17}$ (b) $\frac{17}{\sqrt{15}}$

(c) $\frac{23}{\sqrt{17}}$ (d) $\frac{23}{\sqrt{15}}$

Sol.(c) $\frac{x}{5} + \frac{y}{b} = 1 \qquad \Rightarrow \frac{13}{5} + \frac{32}{b} = 1$

$\Rightarrow \frac{32}{b} = \frac{-8}{5} \qquad \Rightarrow b = -20$

Slope of line $L \Rightarrow -\frac{b}{5} \qquad \Rightarrow m = 4$

Slope of line $K \Rightarrow m' = \frac{-3}{c}$

($\because m = m'$ for parallel lines)

$\Rightarrow -\frac{3}{c} = 4 \qquad \Rightarrow c = -\frac{3}{4}$

$\therefore$ Line $K \Rightarrow -\frac{4x}{3} + \frac{y}{3} = 1$ .....(a)

$\Rightarrow -4x + y = 3 \qquad \Rightarrow 4x - y = -3$

Line $L \Rightarrow \frac{x}{5} + \frac{y}{-20} = 1 \Rightarrow 4x - y = 20$

So distance between line L and K is,

$d = \left|\frac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right| = \left|\frac{-3 - 20}{\sqrt{4^2 + 1^2}}\right| = \frac{23}{\sqrt{17}}$

82. A person is to count 4500 currency notes. Let a_n denote the number of notes he counts in the n^{th} minute. If $a_1 = a_2 = ... = a_{10} = 150$ and $a_{10}, a_{11}, ...$ are in an AP with common difference -2, then the time taken by him to count all notes is

(a) 34 minutes (b) 125 minutes

(c) 135 minutes (d) 24 minutes

Sol.(a) $S = 4500$

$a_1 + a_2 + a_3 + ... + a_{10} = 10(150) = 1500$

Now for remaining notes

$S = 4500 - 1500 = 3000$

$a_{11} = 148, d = -2$ then

$S = \frac{n}{2}\left[2(148) + (n-1)(-2)\right]$

$\Rightarrow 3000 = n(149 - n)$

$\Rightarrow n^2 - 149n + 3000 = 0$

$\Rightarrow n^2 - 125n - 24n + 3000 = 0$

$\Rightarrow n(n - 125) - 24(n - 125) = 0$

$\Rightarrow n = 24$

So minimum time to count currency notes = $24 + 10 = 34$ minute.

83. Let $f : R \to R$ be a positive increasing function with $\lim_{x\to\infty} \frac{f(3x)}{f(x)} = 1$. Then

$\lim_{x\to\infty} \frac{f(2x)}{f(x)} =$

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$

(c) 3 (d) 1

Sol.(d)

84. Let $p(x)$ be a function defined on R such that $p'(x) = p'(1-x)$ for all $x \in [0,1]$, $p(0) = 1$ and $p(1) = 41$. Then $\int_0^1 p(x)\,dx$ equals

(a) 21 (b) 41

(c) 42 (d) $\sqrt{41}$

Sol.(a) $P'(x) = P'(1-x)$

$\int P'(x)dx = \int P'(1-x)dx$

$\Rightarrow P(x) = -P(1-x) + C$ ....(a)

$\Rightarrow P(0) = -P(1) + C$ for $x = 0$

$\Rightarrow 1 + 41 = C \qquad \Rightarrow C = 42$

By (a)

$P(x) = -P(1-x) + 42$

$\Rightarrow \int_0^1 P(x)dx + \int_0^1 P(1-x)dx = \int_0^1 42dx$

$\Rightarrow \int_0^1 P(x) + \int_0^1 P(x)dx = 42(x)_0^1$

$\Rightarrow 2\int_0^1 P(x)dx = 42$

$\Rightarrow \int_0^1 P(x)dx = 21$

85. Let $f:(-1,1) \to R$ be a differentiable function with $f(0) = -1$ and $f'(0) = 1$. Let $g(x) = [f(2f(x)+2)]^2$. Then $g'(0) =$

(a) -4 (b) 0
(c) -2 (d) 4

Sol.(a) $g(x) = [f(2f(x)+2)]^2$

$\Rightarrow g'(x) = 2f(2f(x)+2)\cdot$

$f'(2f(x)+2)(2f'(x))$

$\Rightarrow g'(0) = 2f(2f(0)+2)\cdot$

$f'(2f(0)+2)(2f'(0))$

$\Rightarrow g'(0) = 2f(2(-1)+2)\cdot$

$f'(2(-1)+2)(2(1))$

$\Rightarrow g'(0) = 2f(0)\cdot f'(0)\cdot 2$

$\Rightarrow g'(0) = 4(-1)\cdot(1) = -4$

86. There are two urns. Urn A has 3 distinct red balls and urn B has 9 distinct blue balls. From each urn two balls are taken out at random and then transferred to the other. The number of ways in which this can be done is

(a) 36 (b) 66
(c) 108 (d) 3

Sol.(c) Total no. of ways $= {}^3C_2 \cdot {}^9C_2$

$= {}^3C_1 \cdot {}^9C_2 = 3\dfrac{9.8}{2.1} = 108$

87. Consider the system of linear equations :

$x_1 + 2x_2 + x_3 = 3$

$2x_1 + 3x_2 + x_3 = 3$

$3x_1 + 5x_2 + 2x_3 = 1$

The system has
(a) exactly 3 solutions
(b) a unique solution
(c) no solution
(d) infinite number of solutions

Sol.(c) $x_1 + 2x_2 + x_3 - 3 = 0$

$2x_1 + 3x_2 + x_3 - 3 = 0$

$3x_1 + 5x_2 + 2x_3 - 1 = 0$

$\Rightarrow \Delta = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & 5 & 2 \end{vmatrix} \qquad R_2 \to R_2 + R_1$

$\Rightarrow \Delta = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 5 & 2 \\ 3 & 5 & 2 \end{vmatrix} = 0$

(Two rows are identical)

Now $\Delta x_1 = \begin{vmatrix} 2 & 1 & -3 \\ 3 & 1 & -3 \\ 5 & 2 & -1 \end{vmatrix}$

$R_1 \to R_1 - R_2$

$\Rightarrow \Delta x_1 = \begin{vmatrix} -1 & 0 & 0 \\ 3 & 1 & -3 \\ 5 & 2 & -1 \end{vmatrix}$

$= (-1)(-1-(-6)) = -5 \neq 0$

$\Delta = 0, \Delta x_1 \neq 0 \qquad \Rightarrow$ linear equations has no solution.

88. An urn contains nine balls of which three are red, four are blue and two are green. Three balls are drawn at random without replacement from the urn. The probability that the three balls have different colours is

(a) $\dfrac{2}{7}$ (b) $\dfrac{1}{21}$

(c) $\frac{2}{23}$ (d) $\frac{1}{3}$

Sol.(a) Event A is all three balls are of different colour i.e. 1 Red, 1 Blue, 1 Green

Now, $n(S) = {}^9C_3$

$n(A) = {}^3C_1 \times {}^4C_1 \times {}^2C_1$

So, probaility $= \frac{{}^3C_1 \times {}^4C_1 \times {}^2C_1}{{}^9C_3}$

$= \frac{3 \cdot 4 \cdot 2 \cdot (3)(2)(1)}{9 \cdot 8 \cdot 7} = \frac{2}{7}$

89. For two data sets, each of size 5, the variances are given to be 4 and 5 and the corresponding means are given to be 2 and 4 respectively. The variance of the combined data set is

(a) $\frac{11}{2}$ (b) 6

(c) $\frac{13}{2}$ (d) $\frac{5}{2}$

Sol.(a) We have $\frac{\sum x_1^2}{5} - \left(\frac{\sum x_1}{5}\right)^2 = 4$,

$\frac{\sum x_2^2}{5} - \left(\frac{\sum x_2}{5}\right)^2 = 5$

Also, $\frac{\sum x_1}{5} = 2$,

$\frac{\sum x_2}{5} = 4 \Rightarrow \sum x_1 + \sum x_2 = 30$

$\Rightarrow \frac{\sum x_1^2}{5} = 4 + 4 = 8$,

$\Rightarrow \frac{\sum x_2^2}{5} = 5 + 16 = 21$

$\Rightarrow \sum x_1^2 = 40, \sum x_2^2 = 105$

$\Rightarrow \sum x_1^2 + \sum x_2^2 = 145$

Now, Varience of combined data set

$= \left(\frac{145}{10}\right) - \left(\frac{30}{10}\right)^2$

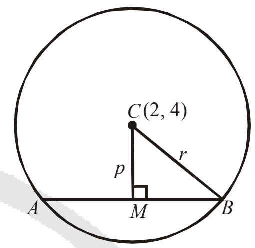
$= 14.5 - 9 = 5.5 = 11/2$

90. The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if

(a) $-35 < m < 15$ (b) $15 < m < 65$

(c) $35 < m < 85$ (d) $-85 < m < -35$

Sol.(a) If circle intersected by a line, then $|p| < r$



$\Rightarrow \left|\frac{3 \times 2 - 4 \times 4 - m}{\sqrt{3^2 + 4^2}}\right| < \sqrt{2^2 + 4^2 + 5}$

$\Rightarrow \left|\frac{6 - 16 - m}{5}\right| < \sqrt{25} \Rightarrow \left|\frac{-10 - m}{5}\right| < 5$

$\Rightarrow |10 + m| < 25$

$\Rightarrow -25 < 10 + m < 25$

$\Rightarrow -35 < m < 15$