PAPER I

Answer key & Solutions

PHYSICS

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
b	b	b	c	c	a	b	a	b	ab	c	bd	b	c	a

16	17	18	
b	a	a	**19.** (A) $\rightarrow$ p,s, (B) $\rightarrow$ r, (C) $\rightarrow$ q, (D) $\rightarrow$ p **20.** (A) $\rightarrow$ r, (B) $\rightarrow$ s, C $\rightarrow$ p, (D) $\rightarrow$ q

CHEMISTRY

21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
c	d	b	d	b	c	b	a	ad	ac	d	bc	d	c	a

36	37	38	
c	c	b	**39.** (A) $\rightarrow$ p,s; (B) $\rightarrow$ p,s; (C) $\rightarrow$ q,s; (D) $\rightarrow$ p,r **40.** (A) $\rightarrow$ r; (B) $\rightarrow$ s; (C) $\rightarrow$ p; (D) $\rightarrow$ q

MATHEMATICS

41	42	43	44	45	46	47	48	49	50	51	52	53	54	55
c	a	b	b	b	c	b	c	abcd	bc	abd	ab	d	c	a

56	57	58	
b	d	b	**59.** (A) $\rightarrow$ q; (B) $\rightarrow$ p; (C) $\rightarrow$ p; (D) $\rightarrow$ r,s **60.** (A) $\rightarrow$ q; (B) $\rightarrow$ s; (C) $\rightarrow$ p,s; (D) $\rightarrow$ r

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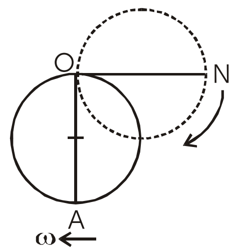
PHYSICS

1.(b) $I =$ Moment of Inertia of disc about axis at $O = \frac{3ma^2}{2}$.



Let the angular speed of disc when OA is vertical be ω.

Rotational kinetic energy of disc

$$= \frac{1}{2} I\omega^2 = \frac{3ma^2\omega^2}{4}$$

Work done against the constant frictional couple $\tau \cdot \theta$

$$= \frac{mga}{2\pi} \cdot \frac{\pi}{2} = \frac{mga}{4}$$

Hence conservation of total energy at N and A gives

$$mga = \frac{3ma^2\omega^2}{4} + \frac{mga}{4}$$

$$\Rightarrow \omega = \sqrt{g/a}$$

2.(b) Since no external torque acts on the system the total angular momentum of the system is conserved. Initially the total angular momentum of man and disc is zero. Let t be the time in which the man walks trhough angle 2π relative to disc. His angular velocity $= \left(\frac{2\pi}{t}\right)$ and his angular momentum relative to disc $= I\left(\frac{2\pi}{t}\right)$.

Where $I = ma^2 =$ moment of inertia of man.

As a result, if the disc (with the man) rotates through the angle ϕ, then its angular velocity $= \left(\frac{\phi}{t}\right)$ and the angular momentum of the system $= \left(I + I_0\right)\frac{\phi}{t}$ where $I_0 = \frac{ma^2}{2}$ is the moment of inertial of the disc. Hence by conservation of angular momentum, $\left(I + I_0\right)\frac{\phi}{t} + I\frac{2\pi}{t} = 0$

$$\Rightarrow -\left(ma^2 + \frac{Ma^2}{2}\right)\phi = \left(ma^2\right)2\pi .$$

Putting $M = 10m$, $-6\phi = 2\pi \therefore \phi = -\frac{\pi}{3}$

3. (b) Let the density per unit area of the disc be ρ.

Area of each hole $= \pi\left(\frac{a}{4}\right)^2 = \frac{\pi a^2}{16}$

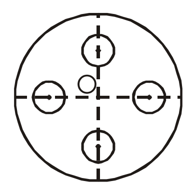
Mass of matter contained in each hole $= \frac{\pi a^2 \rho}{16}$

Mass of matter contained in the disc after the punching of holes

$$= m = \pi\left[a^2 - \frac{a^2}{4}\right]\rho = \frac{3\pi a^2 \rho}{4}$$

Moment of inertia of matter in each hole about an axis through the centre O of disc

$$= I_h = \left(\frac{\pi a^2 \rho}{16}\right)\left[\frac{1}{2}\frac{a^2}{16} + \frac{a^2}{4}\right]$$



$$\Rightarrow I_h = \frac{\pi a^2 \rho}{16}\left[\frac{9a^2}{32}\right]$$

Moment of inertia of the whole disc about its axis

$$= I_0 = \frac{1}{2}\left(\pi a^2 \rho\right)a^2$$

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Hence the required moment of inertia of the given punched disc about $O = I = \left(I_0 - 4 \cdot I_h\right)$

Putting

$$\rho = \frac{4m}{3\pi a^2}, I = \frac{55\pi a^4}{128} \cdot \frac{4m}{3\pi a^2} = \frac{55ma^2}{96}$$

4.(c) $\Delta x = \left(SA + AP\right) - SP$

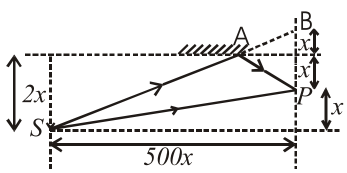
$$= \left(SA + AB\right) - SP$$

$$= SB - SP$$

$$= \sqrt{\left(500x\right)^2 + \left(3x\right)^2} - \sqrt{\left(500x\right)^2 + x^2}$$

$$= 500x\left[\sqrt{1+\frac{9}{500^2}} - \sqrt{1+\frac{1}{500^2}}\right]$$

$$= 500x\left[\left(1+\frac{9}{2\times500^2}\right) - \left(1+\frac{1}{2\times500^2}\right)\right]$$



For maximum at P

$$= \frac{500x}{2\times500^2}\left[8\right] = \frac{\lambda}{2}$$

$$\Rightarrow x = \frac{500}{8}\lambda = 62.5$$

Reflection at A is from denser boundary and condition for maxima $\Delta x = \frac{\lambda}{2}$

5.(c) The shift Δy of the central fringe is given by

$$\Delta y = \frac{\beta}{\lambda}\left(\mu - 1\right)t$$

Where β is the width of the interference fringe.

Here $\Delta y = \beta$

$$\therefore \beta = \frac{\beta}{\lambda}\left(\mu - 1\right)t$$

$$\lambda = \left(\mu - 1\right)t$$

$$\therefore \mu - 1 = \frac{\lambda}{t} = \frac{6\times10^{-5}}{12\times10^{-5}}$$

$$\therefore \mu = 1 + \frac{1}{2} = 1.5$$

6. (a)

7. (b) Intensity of wave $I = \frac{\left(\Delta P\right)_m^2}{2\rho v}$

$$v = \frac{\left(\Delta P\right)_m^2}{2\rho I} = \frac{\left(2\times10^{-4}\right)^2}{2\times1\times10^{-10}} = 200m/s$$

Amplitude of wave

$$A = \frac{\left(\Delta P\right)_m}{\omega\rho v} = \frac{2\times10^{-4}}{10^3\times1\times200} = 10^{-9}m$$

$$\omega = 10^3\, rad/s, k = \frac{\omega}{v} = \frac{10^3}{200} = 5m^{-1}$$

Initial phase $\phi = \frac{\pi}{2}$

The equation of the wave travelling in the negative x-axis

$$y = A\sin\left(\omega t + kx + \phi\right)$$

$$= 10^{-9}\sin\left(1000t + 5x + \frac{\pi}{2}\right)$$

$$= 10^{-9}\cos\left(1000t + 5x\right)$$

8.(a) (Refer to question diagram)

Work done when the mass of 2 kg descends a distance of 1 m along the incline, is

$$W = \left(mg\sin\theta\right)d = 10\sqrt{3}J$$

The extension in the spring is

$$x = 1m$$

The elastic energy stored in the spring is

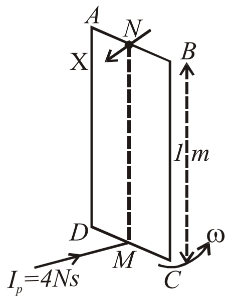
$$\therefore \quad \frac{1}{2}k\,x^2 = 10\sqrt{3}$$

$$\Rightarrow \quad k = 20\sqrt{3} \simeq 35\,N/m$$

Objective questions with one or more than one choice

9. (b)

10.(a,b)



Taking moments of the forces about AB,

$$I_P \times BC = 4 \times 1 = mk^2 \cdot \omega$$

where k is the radius of gyration about edge AB.

For the rectangular lamina, $k^2 = \frac{4a^2}{3}$ where BC =2a=1m is the length of edge BC. Hence

$$mk^2 \cdot \omega = (0.5).$$

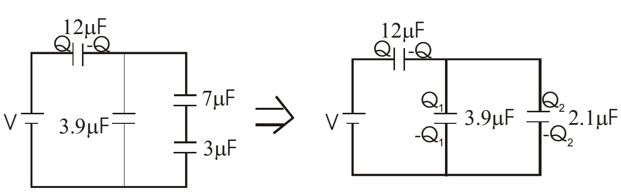
$$\frac{4\left(\frac{1}{2}\right)^2}{3} \cdot \omega = 4 \Rightarrow \omega = 6 \times 4 = 24\, rad/s$$

$$V_{cm} = 24 \times \frac{1}{2} = 12$$

$$MV_{cm} = 6kg\, m/s$$

Impulse at N is along I.

11.(c) Equivalent capacitance of network $= C_{eq} = 4\mu F$



Charge $Q = C_{eq}V = 4V\,\mu C$

The charge on the $7\mu F$ or $3\mu F$ capacitor

$$Q_2 = (7\mu F)(6V)$$

$$= 42\mu C$$

$$\frac{Q_2}{2.1\mu F} = \frac{Q_1}{3.9\mu F}$$

$$\Rightarrow Q_1 = (42\mu C)\frac{(3.9\mu F)}{(2.1\mu F)} = 78\mu C$$

$$Q = Q_1 + Q_2 = (78\mu C + 42\mu C)$$

$$= 120\mu C = 4V\,\mu C$$

emf of the battery $\Rightarrow V = 30\, volt$

The potential drop across

$$12\mu F \text{ capacitor} = \frac{Q}{12\mu F} = \frac{120\mu C}{12\mu F} = 10\, volt$$

12.(b,d)

Let the magnitude of the impulse delivered by the blow be P. If ω be the (positive, clockwise) angular velocity of the rod after the blow and v_c the velocity of the centre of mass, then

$P = Mv_c$ ....(1) and

$I\omega = P(l/2)$ ....(2)

where $I = \frac{Ml^2}{12}$ is the moment of inertia of the rod about its axis through C.

From (1) and (2) , $v_c = \frac{l\omega}{6}$

Now, $l = 1m$ and $\omega = 3rad/s$.

Hence $v_c = \frac{3}{6} = 0.5\, m/s$

For a point P on the rod distant x from A, the acting v acting to the right is given by

$$v = v_c + \omega\left(\frac{l}{2} - x\right)$$

If the point P be in instantaneous rest, $v = 0$. This gives

$$-\omega\left(\frac{l}{2} - x\right) = v_c = \frac{l\omega}{6}$$

Hence $x = \frac{l}{2} + \frac{l}{6} = \frac{2l}{3}$

Hence the point P distant $\frac{2l}{3}$ from A will be at rest after the blow.

13.(b) Shape of the pulse at t = 0

That is a triangular pulse
Area of the pulse

$$= \frac{1}{2}[(4 \times 1) + (1 \times 1)] = \frac{5}{2}\ \text{cm}^2$$

14.(c) $v = \sqrt{\frac{T}{\mu}} = 10$ m/s

Solution of the wave equation that gives displacement of any piece of the string at any time

$$y = f(x,t) = \begin{cases} \frac{(x - vt)}{4} + 1 & \text{for} \quad vt - 4 < x \le vt \\ -(x - vt) + 1 & \text{for} \quad vt < x < vt + 1 \\ 0 & \text{otherwise} \end{cases}$$

Using v = 1000 cm/s, t = 0.01 s
vt = 10 cm
as (vt – 4) < (x = 7 cm) < vt

$$y = \frac{1}{4}(7 - 10) + 1 = \frac{1}{4}\text{cm} = 0.25\ \text{cm}$$

15.(a) Transverse velocity = $\frac{\partial y}{\partial t}$
at t = 0.015 s, vt = 15 cm
as for x = 13 cm (vt – 4) < x < vt
therefore

$$\frac{\partial y}{\partial t} = -\frac{v}{4} = -250\ \text{cm/s}$$

16.(b) When it hits ground displacement in y direction will be zero.

$$5t - \frac{1}{2}15t^2 = 0 \Rightarrow t = \frac{2}{3}\sec$$

17.(a) At time $t = \frac{2}{3}$ sec the displacement in x direction

$$x = 5\sqrt{3}\left(\frac{2}{3}\right) - \frac{1}{2}5\sqrt{3}\left(\frac{2}{3}\right)^2 = \frac{20}{3\sqrt{3}} = 3.85m$$

18.(a) After impact the surface, velocity along vertical direction

$$u'_y = eu_y = 0.2 \times 5 = 1\,m/s$$

Velocity along the horizontal direction after time T_1

$$u'_x = u_x + a_x T_1 = 5\sqrt{3} - 5\sqrt{3} \times \frac{2}{3} = \frac{5}{\sqrt{3}}\,m/s$$

The ball rebounds in the vertical direction with velocity 1 m/s.

19. **(A) $\rightarrow$ p,s, (B) $\rightarrow$ r, (C) $\rightarrow$ q, (D) $\rightarrow$ p**

20. **(A) $\rightarrow$ r, (B) $\rightarrow$ s, C $\rightarrow$ p, (D) $\rightarrow$ q**

Velocity of centre of the mass of system

$$v_m = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

$$= \frac{10 \times 10^{-6} \times 300 + 20 \times 10^{-6} \times 600}{\left(10 \times 10^{-6} + 20 \times 10^{-6}\right)}$$

$$= 500\,m/s$$

Relative energy of the system

$$= \frac{1}{2}\mu v_{rel}^2 + \frac{q_1 q_2}{4\pi\varepsilon_0 r}$$

$$= \frac{1}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right)(v_1 - v_2)^2 + \frac{q_1 q_2}{4\pi\varepsilon_0 r}$$

$$= \frac{1}{2}\left(\frac{10^{-5} \times 2 \times 10^{-5}}{3 \times 10^{-5}}\right)(300)^2 + \frac{1(-5) \times 10^{-12} \times 9 \times 10^9}{10^{-2}}$$

$$= 0.3 - 4.5 = -4.2\,J$$

If the particles do not escape, then the relative energy of the system is equal to electrostatic energy at maximum separation r

$$-4.2J = \frac{q_1 q_2}{4\pi\varepsilon_0 r'} = \frac{-45 \times 10^{-12} \times 10^9}{r'}$$

$$r' = \frac{45 \times 10^{-3}}{4.2} = \frac{15}{14} \times 10^{-4}$$

$$= 10.7 \times 10^{-3}\,m = 10.7\,mm$$

Initial electrostatic potential energy $= -4.5\,J$

C H E M I S T R Y

21.(c) $A(g)+2B(g) \rightleftharpoons C$

n	4	0	t = 0
n-1	4-2	1	equilibrium

$$K_C = 0.25 = \frac{[C]}{[A][B]^2} \Rightarrow [A] = \frac{[C]}{[B]^2(0.25)}$$

$$\frac{n-1}{5} = \frac{1}{5\times 0.25\times 2^2} = 25$$

$$\Rightarrow n = 26 \text{ moles Ans.}$$

22. (d)

23. (b)

24. (d) HA + NaOH $\longrightarrow$ NaA + H_2O

$7 = pK_a + \log \frac{4}{1} \quad \Rightarrow \quad pK_a = 6.4$

$\Rightarrow \quad K_a = 10^{-6.4} = 4 \times 10^{-7}$

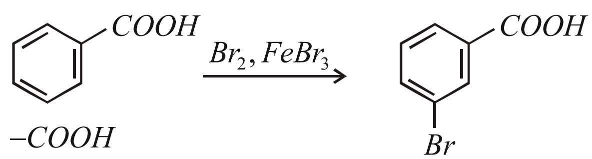
25. (b) $Ag_2SO_4 \rightleftharpoons 2Ag^+ + SO_4^{-2}$

$2s' \quad (s' + 10^{-2})$

$\approx 10^{-2}$

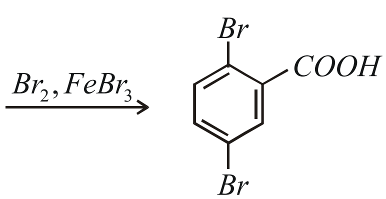
$K_{sp} = (2s')^2 (10^{-2}) = (2 \times 2 \times 10^{-8})^2 (10^{-2}) = 16 \times 10^{-18}$

26. (c)

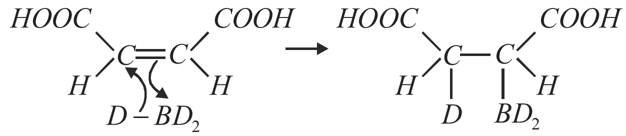


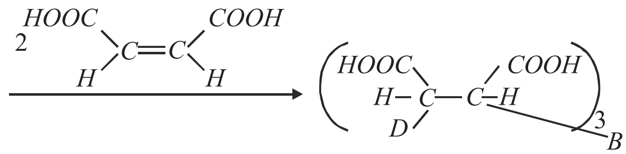
$-COOH$
meta directing

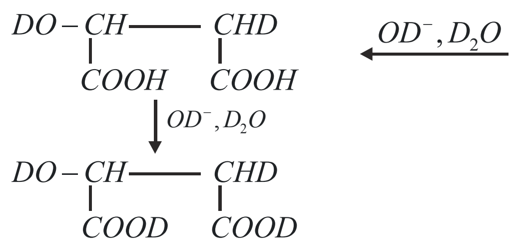
Bromine is deactivating and ortho-para directing. Bromination is favoured at para to -Br (adjacent to -COOH)



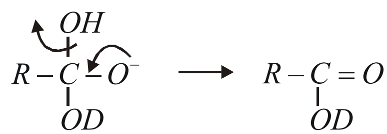
27. (b)





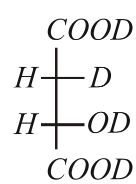


Hydrolysis starts that remain in equilibrium



$cis + syn$ addition $\longrightarrow$ *meso* form or **Erythro**

Hence,



28.(a) $X_{N_2}+X_{H_2}=1-0.1-0.1=0.8$

$X_{N_2}=\frac{1}{4}\times 0.8 \quad \Rightarrow 0.2$

$X_{H_2}=\frac{3}{4}\times 0.8 \quad \Rightarrow 0.6$

$P_{NH_3}=0.1\times 10=1\,\text{atm}$

$P_{N_2}=0.2\times 10=2\,\text{atm}$

$P_{H_2}=0.6\times 10 \quad \Rightarrow 6\,\text{atm}$

$K_P=2\times 216$

$K_P=432(atm)^2$ Ans.

29.(a,d)

30.(a,c) Increase in solvent polarity, increases the life-time of carbocation and stability of carbocation facilitates longer attack from both face without hinderance, hence racemisation more. Moreover solvent polarity decreases the ion-pair interaction also.

31. (d) $HClO_4$ is strong acid.

32. (b,c)

As ΔG^0 for $N_2O_4\rightarrow 2NO_2$ is $+5.0kJ\,mol^{-1}$ (positive) that's why direct conversion is non-spontaneous.

But at equilibrium point B, N_2O_4 is dissociated upto equilibrium point (dissociation of that extent) spontaneous as $\Delta G^0=-0.86kJ$

Again, formation of N_2O_4 upto equilibrium point is fairly spontaneous

$\Delta G^0=-5.40+(-0.84)=-6.24kJ$

Hence, dynamic conversion N_2O_4 into equilibrium mixture and $2NO_2$ into equilibrium mixture both are spontaneous. But, as for the reverse process ΔG^0 is $-6.24kJ$ (more than forward), that's why more favourable.

33.(d) $2NaOH+Sn(OH)\rightarrow Na_2\left[Sn(OH)_6\right]$

Now it forms the $\left[Sn(OH)_6\right]^{2-}$ i.e. negatirely charged sol. Hence it has minimum coagulating value in nighly +rely charged colloidal solution.

34.(c) Total volume after addition of 1 ml of $1\times 10^{-4}\,MBaCl_2$

$=9+1=10ml\cdot$

thus molarity of $BaCl_2$

$BaCl_2=\frac{M_1V_1}{V_{total}}=\frac{10^{-4}\times 1}{10}=10^{-5}$

Now concentratinin millimole per litre

$=10^{-5}\times 1000$

$=10^{-5}\,m\,mole/litre$ of Ba^{2+}

35.(a) Factual.

36(c) given $K_e=4$

$\underset{1}{H_2}+\underset{3}{I_2}\rightleftharpoons\underset{0}{2HI}$

$\frac{1-x}{2} \quad \frac{3-x}{2} \quad x$

$K_c=4=\frac{x^2}{(1-x/2)(3-x/2)}$

$4(1-x/2)(3-x/2)=x^2$

$4\left[3-x/2-3x/2+x^2/4\right]=x^2$

$12-8x+x^2-x^2$

$\Rightarrow \quad 12-8x=0$

$x=1.5$

37(c) We know that

$K_p=K_c(RT)^{\Delta n}$

$=K_c(RT)^0$

$K_p=K_c=41$

38. (b) $2SO_2(g)+O_2(g)\rightleftharpoons 2SO_3(g)$

Initial	2	1.5	0
At equilibrium	$2-2x$	$1.5-x$	$2x$

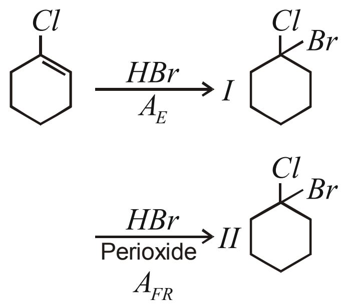
$\therefore (2-2x)\times 2=0.4\times 5$

$$x = 0.5$$

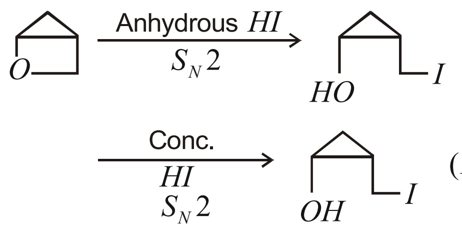
$$K_C = \frac{[SO_3]^2}{[SO_2]^2[O_2]} = \frac{(1/5)^2}{(1/5)^2(1/5)} = 5 \ Ans.$$

39. (A) $\rightarrow$ p,s; (B) $\rightarrow$ p,s; (C) $\rightarrow$ q,s; (D) $\rightarrow$ p,r

(A)



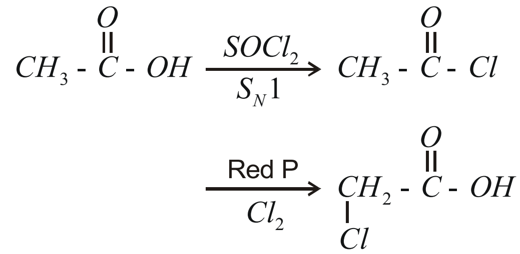
(B)



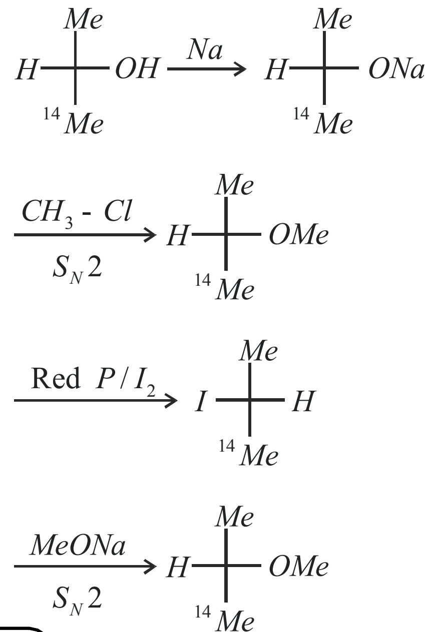
(I)

(II)

(C)



(D)



MATHEMATICS

41. (c) Obviously, f is increasing and g is decreasing.

Hence, f (g (α^2 - 2α)) > f (g (3α - 4))

$\Rightarrow$ g(α^2 - 2α) > g(3α - 4)

$\Rightarrow$ α^2 - 2α < 3α - 4

$\Rightarrow$ $\alpha \in (1, 4)$

42. (d) (i) $f'(x) = 4 + \left(\frac{1}{2} - x\right)^{\frac{-1}{3}}$ in not differentiable at

$x = \frac{1}{2}$ i.e. not in $[0,1]$.

(ii) $g(x)$ is not continuous in $[0,1]$

(iii) $h(0) \neq h(1)$

43. (b) $y = \tan^{-1}\left[\frac{1}{2}\left\{\frac{2\sin x}{|\cos x|}\right\}\right]$

40. (A) $\rightarrow$ r; (B) $\rightarrow$ s; (C) $\rightarrow$ p; (D) $\rightarrow$ q

$y = \tan^{-1}\left(\frac{\sin x}{-\cos x}\right) \qquad \left[\because x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)\right]$

$y = \tan^{-1}(-\tan x)$

$y = \tan^{-1}\{\tan(-x)\}$

$\Rightarrow y = -x \qquad \Rightarrow \frac{dy}{dx} = -1$

44. (b) $y = \log_{e^x}(x-2)^2$

$\Rightarrow y = \frac{\log_e (x-2)^2}{\log_e e^x}$

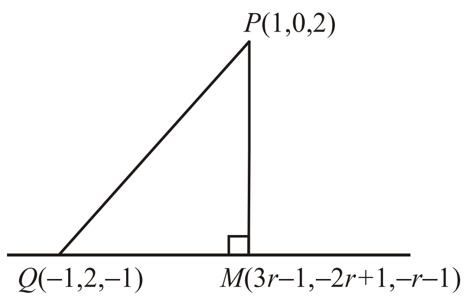
can be written as $y(x) = \frac{2\log_e(x-2)}{x}$

$$\Rightarrow y'(x) = 2\left[\frac{x\dfrac{1}{x-2} - \log_e(x-2)}{x^2}\right]$$

$$\Rightarrow y'(x) = 2\left[\frac{x-(x-2)\log_e(x-2)}{(x-2)x^2}\right]$$

$$\Rightarrow y'(3) = 2\left(\frac{3}{9}\right) = \frac{2}{3}$$

45. (b) $\dfrac{x+1}{3} = \dfrac{y-2}{-2} = \dfrac{z+4}{-1} = r$



$\therefore\ M(3r-1, -2r+2, -r-1)$

DR's of PM are $3r-2, -2r+2, -r-3$

DR's of line QM are $3, -2, -1$

$\because\quad PM \perp QM$

$\therefore\ 9r-6+4r-4+r+3=0$

$\Rightarrow 14r-7=0 \Rightarrow r=\dfrac{1}{2}$

$\therefore\ M\left(\dfrac{1}{2}, 1, -\dfrac{3}{2}\right)$

46. (c) Equation of AB $\equiv \dfrac{x-3}{0} = \dfrac{y-4}{6} = \dfrac{z+5}{-18}$

Equation of CD $\equiv \dfrac{x-1}{4} = \dfrac{y-2}{0} = \dfrac{z-5}{-8}$

For any point of line AB & CD to be common following three equations must be consistent.

$4t_2 + 1 = 3$ …(i)

$-8t_1 + 5 = -18t_2 - 5$ …(ii)

$2 = 6t_1 + 4$ …(iii)

that is true for $t_2 = ½$ & $t_1 = -1/3$

Hence, point of intersection P of AB & CD is (3, 2, 1)
Now for hitting each other.

$$\frac{AP}{V_1} = \frac{CP}{V_2}$$

$$\Rightarrow \frac{V_1}{V_2} = \frac{AP.}{CP.} = \frac{\sqrt{0+4+36}}{\sqrt{4+0+16}} = \frac{\sqrt{40}}{\sqrt{20}} = \sqrt{2}$$

47. (b) First common term of the sequences 7,13,19 and 3,17,31 is 31 and common difference of the series of common terms is the LCM of common differences of the given series i.e. 42. Now, sum of n common terms is $\dfrac{n}{2}\left[2(31)+(n-1)42\right]$.

48. (c) For $2 \le n \le 9, S_n - S_{n-1} = \underbrace{nn.....n}_{n\text{ times}} = \dfrac{n}{9}\left(10^n - 1\right)$

$$\Rightarrow 9(S_n - S_{n-1}) = n\left(10^n - 1\right)$$

49. (a,b,c,d)
We have

$$\sum_{r=1}^{n} r(r+1)(2r+3) = \sum_{r=1}^{n}\left(2r^3+5r^2+3r\right)$$

$$= 2\sum_{r=1}^{n} r^3 + 5\sum_{r=1}^{n} r^2 + 3\sum_{r=1}^{n} r$$

$$= 2\left[\frac{1}{4}n^2(n+1)^2\right] + \left[\frac{1}{6}n(n+1)(2n+1)\right] + 3\left[\frac{1}{2}n(n+1)\right]$$

$$= \frac{1}{2}n^4 + \frac{8}{3}n^3 + \frac{9}{2}n^2 + \frac{7}{3}n$$

$\therefore\ a = 1/2,\ b = 8/3,\ c = 9/2,\ d = 7/3$

and $e = 0$

50. (b,c)

$$f(x) = 1 + x + x^2 + ... + x^n$$

$$\therefore\ f'(x)\cdot g(x) = \left(1 + 2x + 3x^2 + ... + nx^{n-1}\right)$$

$$\times\left\{1 - \frac{2}{x} + \frac{3}{x^2} - ... + (-1)^n \cdot \frac{(n+1)}{x^n}\right\}$$

$\therefore$ the required constant term

$=1-2^2+3^2-...+(-1)^{n-1}n^2$

Separate the case of n, odd or even.

(i) If n is odd, then sum = $1+2+3+....n$ term

(ii) If n is even, then sum = $-1-2-3-....-n$ term

51. (a,b,d)

We have the lines $x+y=5, 7x-y=3$

Now BC is equally inclined with the lines of slopes (-1) & 7 respectively

$$\Rightarrow \frac{m-(-1)}{1m(-1)}=\frac{-(m-7)}{1+m(7)}$$

$$\Rightarrow (m+1)(7m+1)=(m-1)(m-7)$$

$$\Rightarrow m=\frac{1}{3},3$$

So, the lines may be $x-3y=\lambda$

and $3x+y=\mu$.

We find the intersection point of lines $x+y=5$, $7x-y=3$ and $x-3y=\lambda$.

Also area of triangle is 5 (given) the find the value of λ.

Similarly, we find the value of μ

52. (a,b)

(i) $f'(x)=\frac{1}{|x|}=\frac{1}{x}, x\in(0,\infty)\Rightarrow f'(x)>0$

(ii) $g'(x)=-\frac{1}{x^2}<0,$ so $g(x)$ decreases on every interval in its domain

(iii) $h(x)=|\ln x|$ is increasing in $(1,\infty)$ not in $(0,\infty)$

53. (d) We have, lines

$$\cos 2\theta(2x+3y-5)+\sin^2\theta(2x-4y+2)=0$$

which are passes through the intersection of lines

$2x+3y-5=0$

and $2x-4y+2=0$ i.e. (1, 1).

54. (c) Now, the line of above family, which is farthest from origin is perpendicular the line joining the points (0, 0) & (1, 1)

i.e. slope of the line is - 1.

Since it passes through the point (1, 1) then equation of the line is $x+y=2$.

Also, coordinate of perpendicular foot drawn from (0, 0) to $x+y=2$ is (1, 1).

If reflection of origin in the line is (α,β) then

$$1=\frac{0+\alpha}{2}, 1=\frac{0+\beta}{2} \quad \text{i.e. } (\alpha,\beta)=(2,2)$$

55. (a) The line of above family, which is nearest from origin is the line joining the point (0, 0) & (1, 1) i.e. $x=y$. Now, intersection points of $7x+y+16=0$ with $x-y=0$ and $x+y=2$ are $(-2,-2)$ & $(-3,5)$ respectively.

So the area of the triangle

$$=\frac{1}{2}\left[1(-7)+(-2)(4)+(-3)(3)\right]=12$$

(neumerically).

56. (b) $f'(x)=3(5)x^4-5(4)x^3$, for $f'(x)=0$ extermum

$$\Rightarrow 5x^3(3x-4)=0 \Rightarrow x=0,\frac{4}{3}$$

Also $f''\left(\frac{4}{3}\right)=+ve$, $f''(0)=0$, $f'''(0)=0$ but $f^{iv}(0)=-ve$

So, function has maxima at $x=0$ and minima at $x=\frac{4}{3}$

57. (d) $f'(x)=5x^3(3x-4)$ is increasing function in interval $(-1,0)$, decreasing function in interval $\left(0,\frac{4}{3}\right)$ and increasing function in interval $\left(\frac{4}{3},\frac{3}{2}\right)$.

Also. $f(-1)=-8$, $f(0)=0$, $f\left(\frac{4}{3}\right)=\frac{-256}{81}$,

$f\left(\frac{3}{2}\right)=\frac{-181}{32}$

So difference of greatest an least value of function is 8.

58. (b) In this case, rate of change of $f'(x)$ changes its sign when $f''(x)=0$, which is for $x=1$. So, value of function at the point $x=1$ is $f(1)=-2$.

59. (A)$\rightarrow$q; (B)$\rightarrow$p; (C)$\rightarrow$p; (D)$\rightarrow$r,s

(A) Equation of the line passes through the point (1, -2, 3) and parallel to the line $\frac{x}{2}=\frac{y}{3}=\frac{z}{-6}$ is

$\frac{x-1}{2}=\frac{y+(2)}{3}=\frac{z-3}{-6}=\mu$ (say) (1)

$\Rightarrow x=2\mu+1, y=3\mu-2, z=-6\mu+3$ is a point on line.

To find intersection point of line (1) with plane $x-y+z=\lambda$

the point of line lie on plane i.e.

$2\mu+1-3\mu+2-6\mu+3=\lambda \Rightarrow \mu=\frac{6-\lambda}{7}$ ...... (2)

Now the distance of point (1, -2, 3) with the intersection point

$=\sqrt{(2\mu)^2+(3\mu)^2+(-6\mu)^2}$

$=7\mu=6-\lambda=\pm 1$

(given) $\Rightarrow \lambda=5 \ (\lambda>0)$

(B) Direction ratio of diagonal of a cube is $1,1,1$ and direction ratio of diagonal of one face $1,1,0$ (in xy-plane). So, angle between the diagonal of a cube and a diagonal of one face $=\cos^{-1}\sqrt{\frac{2}{3}}$

(C) Line $\frac{x-1}{-2}=\frac{y-(-1)}{1}=\frac{z-3}{-4}$

Passes through the point $(1,-1,3)$. Since line lies in the plane i.e. point $(1,-1,3)$ lies on the plane $2x-z+\lambda=0 \Rightarrow 2-3+\lambda=0$

$\Rightarrow \lambda=1$

(D) Let the given points be A and C respectively. Then

$AB^2=350, AC^2=500-20\lambda+\lambda^2,$

$BC^2=150+10\lambda+\lambda^2$

Now, $AB^2+AC^2=BC^2$

$\Rightarrow 350+500-20\lambda+\lambda^2$

$=150+10\lambda+\lambda^2 \Rightarrow \lambda=\frac{70}{3}$

Next, $AB^2+BC^2=AC^2$

$\Rightarrow 350+150+10\lambda+\lambda^2$

$=500-20\lambda+\lambda^2$

$\Rightarrow \lambda=0$, has no real value of λ

Further, $BC^2+AC^2=AB^2$

$\Rightarrow 150+10\lambda+\lambda^2+500-20\lambda+\lambda^2=350$

$\Rightarrow \lambda^2-5\lambda+150=0$

60. (A)$\rightarrow$q; (B)$\rightarrow$s; (C)$\rightarrow$p,s; (D)$\rightarrow$r

(A) $A=G+\frac{3}{2}$ and $G=H+\frac{6}{5}$

$\because G^2=AH \Rightarrow g^2=\left(G+\frac{3}{2}\right)\left(G-\frac{6}{5}\right)$

$\Rightarrow G=6$, $A=\frac{15}{2}$

$\Rightarrow \lambda=12$, $\mu=3 \Rightarrow \lambda+2\mu=18$

(B) $\because 4y\frac{dy}{dx}=2\lambda x$

$\Rightarrow \left.\frac{dy}{dx}\right|_{(1,-1)}=\frac{\lambda}{-2}=-1 \Rightarrow \lambda=2$

Also $(1,-1)$ lies on $2y^2=\lambda x^2+\mu$

$\Rightarrow 2=\lambda+\mu$

$\therefore \lambda=2, \mu=0 \Rightarrow \lambda-\mu=2$

(C) $\because\ f(x) = x^2 - 2|x| = \begin{cases} x^2 - 2x, & x \geq 0 \\ x^2 + 2x, & x < 0 \end{cases}$

$\therefore\ f'(x) = \begin{cases} 2x - 2, & x > 0 \\ 2x + 2, & x < 0 \end{cases}$

Clearly $f'(x)$ is not defined at $x = 0$ and $f'(x) = 0$ at $x = \pm 1$ i.e. $x = -1, 0, 1$ are critical points

$\therefore\ \lambda = 1, \mu = -1$

$\Rightarrow 3\lambda + \mu = 2$, $\lambda - \mu = 2$

(D) The function $f(x)$ is differentiable except at $x = 0$

$\therefore\ f(x)$ is not continuous at $x = 0$, also

$\because\ f(x) = 2x^2 + \frac{2}{x^2} \Rightarrow f'(x) = 4x - \frac{4}{x^3}$

Critical points are $x = 1, -1$

Now, values of $f(x)$ at $x = -2, -1, 0, 1, 2$

are $f(-2), f(-1), f(0), f(1), f(2)$

ie., $\frac{17}{2}, 4, 1, 14, \frac{17}{2}$

$\Rightarrow\ \lambda = \frac{17}{2}, \mu = 1$

ALL INDIA OPEN TEST FOR IIT-JEE 2010 ON 31st Jan,