PAPER II

Answer key & Solutions

P H Y S I C S

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
a	d	c	b	abc	abcd	abc	ad	abcd	4	2	1	4	3	1

16	17	
1	5	**18.** (A) $\rightarrow$ p,q,r; (B) $\rightarrow$ p,q,r; (C) $\rightarrow$ q,s; (D) $\rightarrow$ p,s **19.** (A) $\rightarrow$ p,r; (B) $\rightarrow$ q; (C) $\rightarrow$ q; (D) $\rightarrow$ s

C H E M I S T R Y

20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
d	d	d	c	acd	bcd	d	abc	c	5	4	4	2	8	1

35	36	
4	6	**37.** (A) $\rightarrow$ p; (B) $\rightarrow$ q,s; (C) $\rightarrow$ q,r; (D) $\rightarrow$ r,s **38.** (A) $\rightarrow$ p; (B) $\rightarrow$ q; (C) $\rightarrow$ s; (D) $\rightarrow$ r

M A T H E M A T I C S

39	40	41	42	43	44	45	46	47	48	49	50	51	52	53
c	d	d	b	ac	abc	abcd	bd	abd	2	3	2	1	4	2

54	55	
2	4	**56.** (A) $\rightarrow$ r; (B) $\rightarrow$ s; (C) $\rightarrow$ r; (D) $\rightarrow$ p,q,r **57.** (A) $\rightarrow$ s, (B) $\rightarrow$ r, (C) $\rightarrow$ q, (D) $\rightarrow$ q

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P H Y S I C S

1.(a) $X = \lambda N$

$$X = \frac{\ln 2}{Y} N$$

$$\Rightarrow \quad N = \frac{XY}{\ln 2}$$

2.(d) For pure rolling

$V = WR$

$\Rightarrow \quad 3 = W \times 0.3$

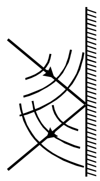
$\Rightarrow \quad W = 10\ rad/\sec$

Speed of point B is

$V_B = 3 - wr$

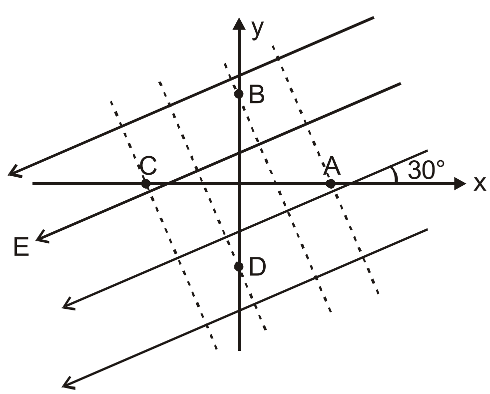
$= 3 - 10 \times 0.1$ = $2\, m/s$

3.(c)



The wave fronts are always perpendicular to the light rays.
Hence, (c).

4.(b) Four lines, perpendicular to lines of electric field and passing through A, B, C and D are drawn. These are equipotential lines. As potential decreases in the direction of electric field, therefore $V_A > V_B > V_D > V_C$



5. (a,b,c)
6. (a,b,c,d)

7. (a,b,c)

Charge on $a_1 = (r_1\,\theta)\,\lambda$
Charge on $a_2 = (r_2\,\theta)\,\lambda$

Ratio of charges = $\frac{r_1}{r_2}$

E_1, Field produced by $a_1 = \frac{K[(r_1\,\theta)\lambda]}{r_1^2}$

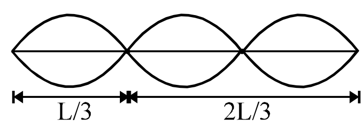
$$= \frac{K\theta\lambda}{r_1}$$

E_2, Field produced by $a_2 = \frac{K\theta\lambda}{r_2}$

as $r_2 > r_1$
Therefore $E_1 > E_2$
i.e. Net field at A is towards a_2.

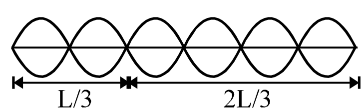
$V_1 = \frac{K.(r_1\theta)}{r_1} = K\theta\lambda$

8.(a,d) $\frac{3\lambda}{2} = L$



Minimum loops = 3

Fundamental $v = \frac{3}{2L} V$



Next higher loops = 6
Ans. : (a, d)

9. (a,b,c, d)

(a) $47.22 = z^2 \times 13.6 \times \left[\frac{1}{2^2} - \frac{1}{3^2}\right]$

$\Rightarrow z = 5$

(b) $E_\infty - E_1 = 13.6 \times 5^2 \left[\frac{1}{1^2} - \frac{1}{\infty^2}\right] = 13.6 \times 25\ eV$

$$\lambda = \frac{hc}{\Delta E} = \frac{1242eV - nm}{13.6 \times 25eV} = 3.653nm$$

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(c) $r = \frac{a_0.n^2}{Z} = \frac{53\times10^{-12}}{5}\times1^2 = 10.6\,pm$

(d) Angular momentum

$= \frac{nh}{2\pi} = 1.05\times10^{-34}\,J-s$

10. F r – *f* r = 0

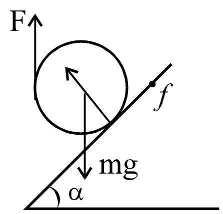
mg sin α – F sin α – *f* = 0

F = *f* = $\frac{mg\sin\alpha}{1+\sin\alpha}$

F cos α + N – mg cos α = 0

N = $\left(mg - \frac{mg\sin\alpha}{1+\sin\alpha}\right)\cos\alpha$

N = $\frac{mg\cos\alpha}{1+\sin\alpha}$



$f_{max} = \mu N$

$\frac{mg\sin\alpha}{1+\sin\alpha} = \frac{\mu mg\cos\alpha}{1+\sin\alpha}$

μ = tan α = 0.75 **[ANS : 4]**

11. Applying COE

mgs (sin α – sin β) = $\frac{1}{2}$ I ω² + 2 $\left(\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2\right)$ where $\omega = \frac{v}{r}$ and

I = $\frac{mr^2}{2}$

Putting values and solving

m g s (sin α – sin β) = $\frac{7}{4}$ m v²

∴ v = $2\sqrt{\frac{1}{7}gs(\sin\alpha - \sin\beta)}$

$= 2\sqrt{\frac{10\times3.5}{7}\left(\frac{4}{5}-\frac{3}{5}\right)}$ = 2 m/s

Ans. : 2

12. $y = A\sin kx\cos\omega t$

$5\sqrt{3} = A\sin k\times10$

$5\sqrt{3} = A\sin k\times20$

after solving $k = \frac{\pi}{30}$

$5\sqrt{3} = A\sin\left(\frac{\pi}{30}\times10\right) \Rightarrow A = 10\,mm$

= 1 cm

Ans . 1

13. $420 = \frac{n}{2l}\sqrt{\frac{T}{m}}$

and $490 = \frac{n+1}{2l}\sqrt{\frac{T}{m}} \Rightarrow \frac{1}{2l}\sqrt{\frac{T}{m}} = 70$

$\Rightarrow$ $l = \frac{1}{140}\sqrt{\frac{T}{m}}$ = 4

Ans . 4

14. $\frac{13.6Z^2}{4} - \frac{13.6Z^2}{n^2} = 28.10$

$\frac{13.6Z^2}{9} - \frac{13.6Z^2}{n^2} = 11.1$

Solving $Z = 3$

Ans . 3

15. V= 150 m/s

$v_p = \frac{-\delta y}{\delta x}v_w$

$v_p = \frac{1cm}{1.5m}\times 150$ m/s

v_p = –1 m/s (downwards)

Ans . 1

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16. In com frame both the particles will have equal x opposite linear momentum.
$\therefore$ **Ans. 1**

17. In Ist process

$$\Delta U_1 = K\frac{q^2}{\sqrt{5}r}\times 4 - Kq^2\left[\frac{2}{r}+\frac{2}{3r}\right]$$

$$=\frac{Kq^2}{r}\left[\frac{4}{\sqrt{5}}-\frac{8}{3}\right]$$

$$\therefore\ W_{elect} = -\Delta U_1$$

In second process there is no change in pE.
Ans. 5

18. (A)$\to$p,q,r; (B)$\to$p,q,r; (C)$\to$q,s; (D)$\to$p,s
For pure rolling

$$\mu_{\min} = \frac{\tan\theta}{\frac{R^2}{K^2}+1}$$

Object	$\mu_{\min}$
Sphere	$\frac{\tan\theta}{3.5}$
Shell	$\frac{\tan\theta}{2.5}$
Disc	$\frac{\tan\theta}{3}$
Ring	$\frac{\tan\theta}{2}$

(a) Disc and sphere will roll without sliding.
$\therefore$ Energy of Disc & sphere will remain conserved.
(b) Only shell disc and sphere will not slide
(c) All will slide no rotation
(d) All will roll without sliding.

19. (A)$\to$p,r; (B)$\to$q; (C)$\to$q; (D)$\to$s

(a) The charge Q_2 is negative and the charge Q_1 is positive.

(b) Since the electric field is zero at a distance 'a' from the point 2, therefore,

$$\frac{Q_1}{(l+a)^2}-\frac{Q_2}{a^2}=0$$

$$\Rightarrow \frac{Q_1}{Q_2}=\left(\frac{l+a}{a}\right)^2$$

(c) For all points $x > a$

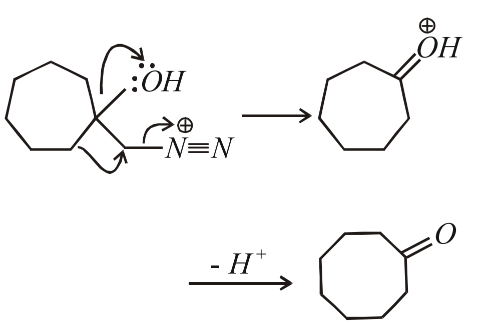
$$E=\frac{Q_1}{(l+x)^2}-\frac{Q_2}{a^2}$$

For max $E, \frac{dE}{dx}=0=-\frac{2Q_1}{(l+x)^3}+\frac{2Q_2}{x^3}$

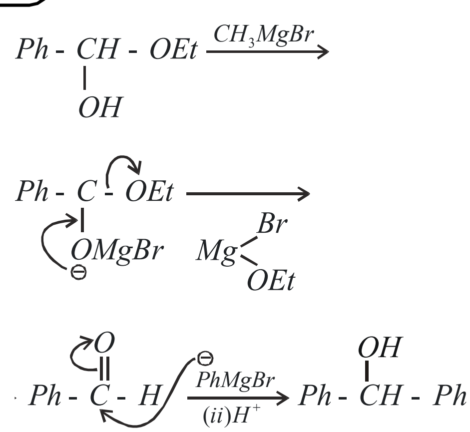
Thus, $x=\frac{1}{\left(\frac{Q_1}{Q_2}\right)^{1/3}-1}=b$

CHEMISTRY

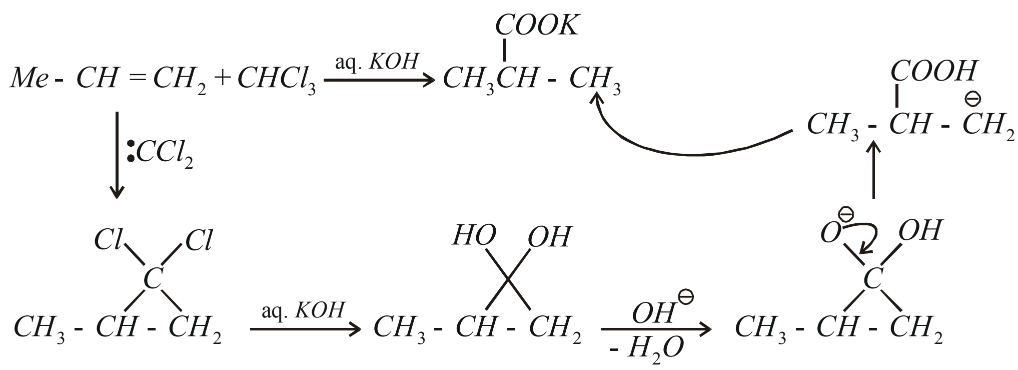
20. (d)

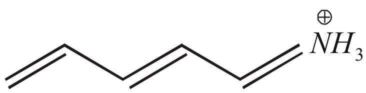


21. (d)



22. (d)



23. (c) 

(N can never be pentavalent)

24. (a,c,d)

(b) 1-pentene give four products with NBS

i.e. $CH_2 = CH - \overset{*}{C}H(Br) - CH_2 - CH_3$ S and R products

(↑ chiral carbon)

$Br - CH_2 - CH = CH - CH_2 - CH_3$ E and Z products

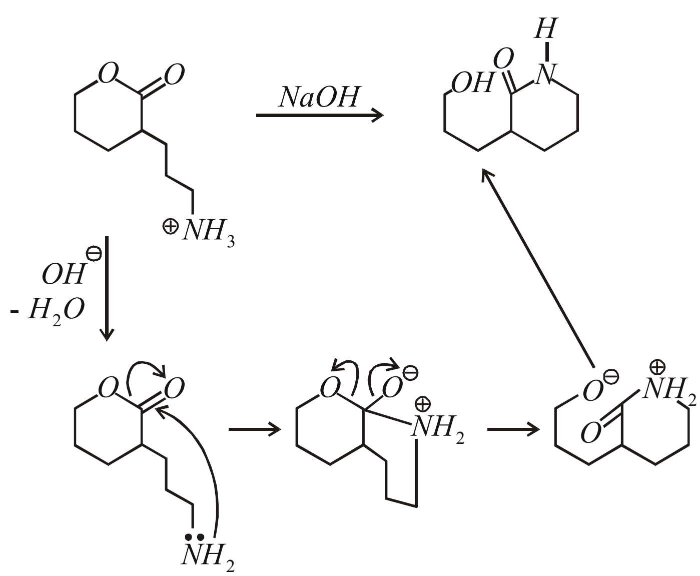
(Stereo centre)

(c) Due to formation of more stable carbocation on increasing phenyl ring at C of $C - X$.

(d) There is no formation of $H^{\bullet}$ during photohalogenation.

25. (b, c, d)

(a)



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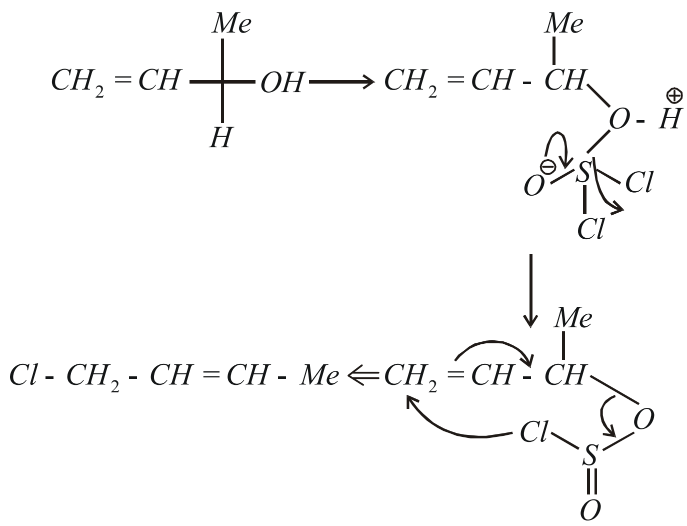
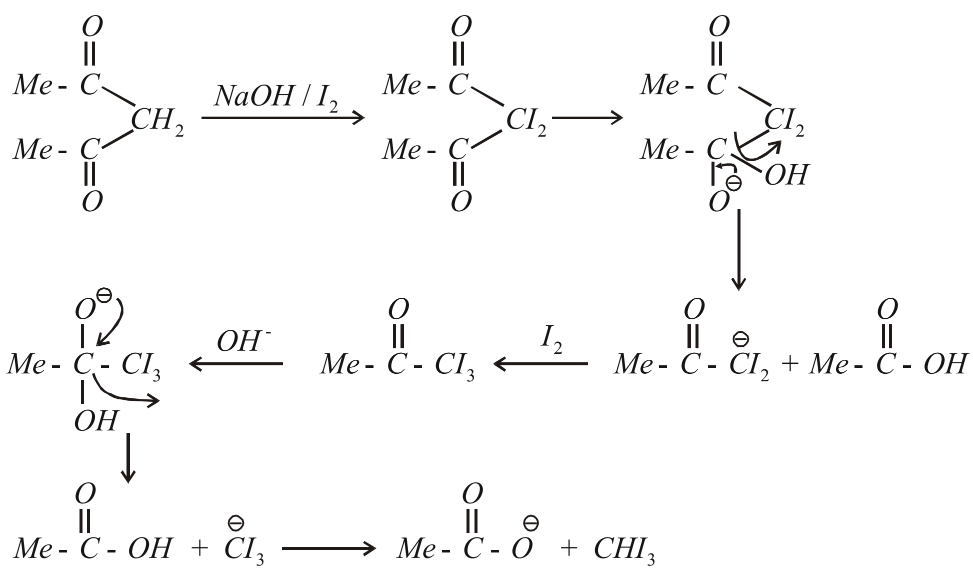
(b)



(c)



So, one mol per mol CHI_3 is formed.

(d)

$$CH_3 - \underset{OH}{\underset{|}{CH}} - CH_3 \xrightarrow{NH_4OH / I_2} \text{no reaction}$$

$NaOH + I_2$ is a good oxidising agent but $NH_4OH + I_2$ is not.

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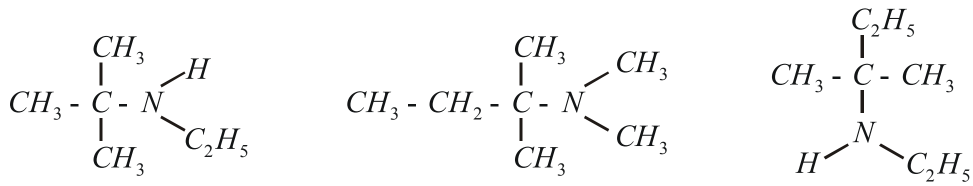
Sol.26. (d)



(I) (II) (III)

$I \,\&\, II \rightarrow$ Functional isomer

$II \,\&\, III \rightarrow$ Not isomer

$I \,\&\, III \rightarrow$ Not isomer

27. (a,b,c)

28. (c) $R_3\overset{+}{N}-CH=CH_2 \xrightarrow{HBr} R_3\overset{+}{N}-CH_2Br$ anti-Markownikoff's

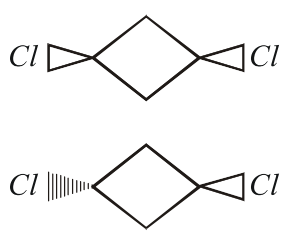
$CH_3-CH=CH_2 \xrightarrow{HBr,\ Peroxide} CH_3-CH_2-CH_2Br$ anti-Markownikoff's

$CH_3-CH=CH-CH_3 \xrightarrow{HBr,\ Peroxide} CH_3-CHBr-CH_2-CH_3$

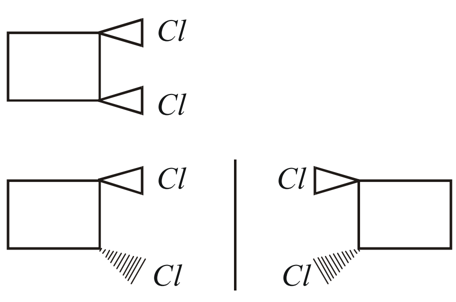
Reactant molecule is symmetrical, here Markownikoff's/anti-Markownikoff's both are meaningless

$CH_3-CH=CH_2$ CCl_3Br , peroxide $CH_3-CHBr-CH_2CCl_3$ anti-Markownikoff's

29. **(5)**



2 Geometrical isomers (a)



2 Geometrical isomers and 3 Optical isomers,
3 Stereo isomers (b)

(a) + (b) = 5 Ans.

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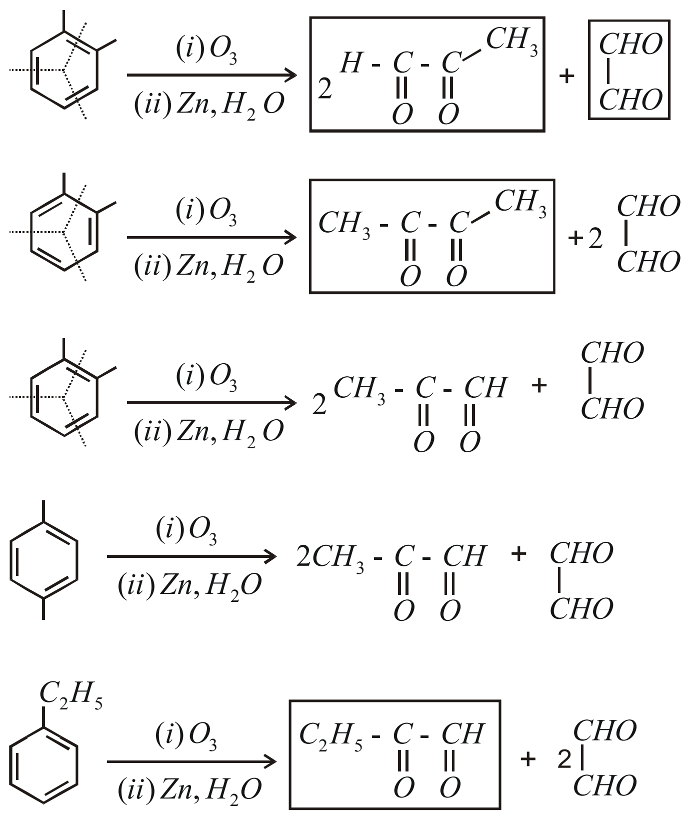
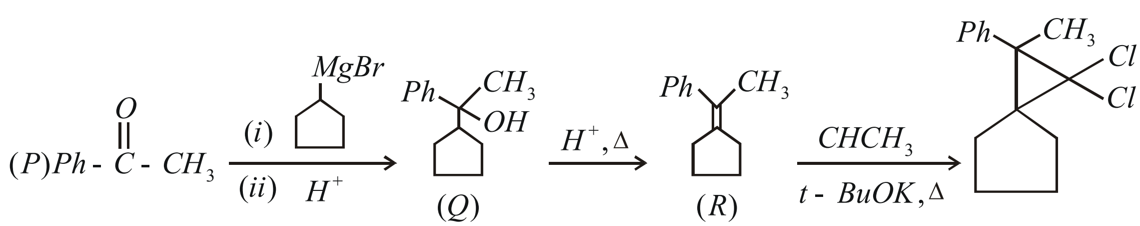
30. **(4)**



so we obtain four types of product (given in bracket) hence total 4 different carbonyl compounds.

31. **(4)**

32. **(2)**



33. **(8)** Balanced equation will be

$3I_2(s) + 6OH^- \rightleftharpoons 5I^-(aq) + IO_3^-(aq) + 3H_2O(l)$

$\Delta G^0 = -172.5 \text{ kJmole}^{-1}$

$$= -\frac{25}{3} \times 300 \times 2.3 \times 10^{-3} \log k$$

$\log k = 30$

$$10^{30} = \frac{10^{-5} \times 10^{-1}}{[OH^-]^6}$$

so $[OH^-] = 10^{-6}$

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34.Sol. $PV = nRT$

$1.013 \times 0.129 = n \times 0.0821 \times 273$

$n = 0.00583$ mole

Area occupied = $0.00583 \times 6.023 \times 10^{23} \times 0.02852 \times 10^{-20}$

= 1 m^2 per gram

35.Sol. Second dissociation constant is very small. H^+ ion concentration mainly depends on the first dissociation constant

$$H_2S \underset{}{\overset{K_1}{\rightleftharpoons}} H^+ + HS^-$$

Hence $\left[H^+\right] = \sqrt{K_1 \times C}$

$= \sqrt{1 \times 10^{-7} \times 0.1}$ = 10^{-4}

pH = 4

36.Sol. $3A(g) + B(g) \rightleftharpoons 2C(g)$

$$K_c = \frac{[C]^2}{[A]^3 . [B]}$$

$$9 = \frac{[2/V]^2}{[2/V]^3 \times [2/V]}$$

$V^2 = 9 \times 4$

$V = 6\, litre$

37. **(A)$\rightarrow$p; (B)$\rightarrow$q,s; (C)$\rightarrow$q,r; (D)$\rightarrow$r,s**

38. **(A)$\rightarrow$p; (B)$\rightarrow$q; (C)$\rightarrow$s; (D)$\rightarrow$r**

(A) $[H^+]_{\text{remaining}} = \dfrac{5 \times 0.08 - 10 \times 0.03}{10 + 5 + 485} = 2 \times 10^{-4}$

$pH = 3.7$

(B) $RHN_3^+ +$ $OH^- \longrightarrow RNH_2 + H_2O$; $[RNH_2] = \dfrac{5}{50} \Rightarrow 0.1$

Initial m.moles 10×0.05 40×0.0125

_____ _______ 0.5 ; $K_{b(RNH_2)} = \dfrac{10^{-14}}{10^{-19}} = 10^{-5}$

$\because \alpha < 0.05$ $\therefore [OH^-] = \sqrt{10^{-5} \times 0.1} \Rightarrow 10^{-3}$

$pOH = 3; pH = 1$

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(C) $pH = pK_{a_2} + \log\left(\frac{0.2}{0.4}\right); pH = pK_{a_2} + \log\frac{[CO_3^{2-}]}{[HCO_3^-]}$

$pH = 10.4 + \log\left(\frac{1}{2}\right) = 10.1$

(D) $Co(OH)_3(s) \rightleftharpoons Co^{3+}(aq) + 3OH^-(aq)$

$3x + 10^{-7}$

$2.7\times10^{-43} = x(3x+10^{-7})^3$ or $2.7\times10^{-43} \simeq x(10^{-7})^3$

$x = 2.7\times10^{-22}$

$[OH^-]_{total} = 3x + 10^{-7} \simeq 10^{-7}$ or $pH \simeq 7$

MATHEMATICS

39. (c) $b-c,\ 2b-x,\ b-a$ are in HP

$\Rightarrow\ 2(b-c)(b-a) = (2b-x)(2b-a-c)$

$\Rightarrow\ 2(b^2-ab-bc+ac) = 4b^2-2ab-2bc-2bx+ax+ac$

$\Rightarrow\ 2ac = 2b^2-2xb+ax+cx$

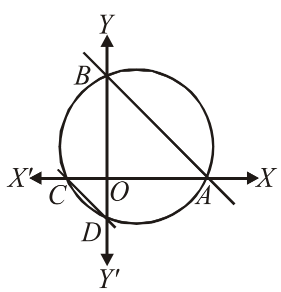
$\Rightarrow\ ac-(x/2)(a+c) = b^2-xb$

$\Rightarrow\ ac-(x/2)(a+c)+x^2/4 = b^2-xb+x^2/4$

$\Rightarrow\ \left(a-\frac{x}{2}\right)\left(c-\frac{x}{2}\right) = \left(b-\frac{x}{2}\right)^2$

$\Rightarrow\ a-\frac{x}{2}, b-\frac{x}{2}$ and $c-\frac{x}{2}$ are in G.P.

40. (d)



If $L_1 \equiv a_1x+b_1y+c_1=0$ meets the co-ordinates axes at A and B and $L_2 \equiv a_2x+b_2y+c_2=0$, meets at C & D. then co-ordinates of A,B,C,D are

$A\left(-\frac{c_1}{a_1},0\right), B\left(0,-\frac{c_1}{b_1}\right), C\left(-\frac{c_2}{a_2},0\right)$

and $D\left(0,-\frac{c_2}{b_2}\right)$

Since A,B,C,D are concyclic, therefore

$OA\cdot OC = OD\cdot OB$

$\Rightarrow\ A\left(-\frac{c_1}{a_1}\right)\left(-\frac{c_2}{a_2}\right) = \left(-\frac{c_2}{b_2}\right)\left(-\frac{c_1}{b_1}\right)$

$\Rightarrow\ a_1a_2 = b_1b_2$

41. (d)

(i) $f'(x) = 3x^2-3$

Now, $f'(x)<0$ for $x\in[0,1)$ and $f'(x)=0$ for $x=1$

$\Rightarrow f(x)$ is decreasing function for a suitable value of m. So, it can have only one root of $f(x)=0$

(ii) $f'(x) = \frac{2}{(x+2)^2}$ So, $f'(c) = \frac{f(4)-f(0)}{4-0}$

$\Rightarrow \frac{2}{(c+2)^2} = \frac{2}{4(3)}$ $\Rightarrow c^2+4c-8=0$

$\Rightarrow c = -2 \pm 2\sqrt{3}$

$\Rightarrow c = 2\left(\sqrt{3}-1\right) \in (0,4)$

(iii) Given function $f(x) = \dfrac{x+1}{x-1}$ is not continuous in $[0,2]$ so, there is no value of c.

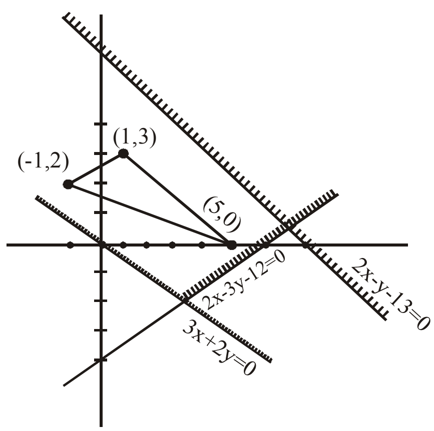
42. (b) $\dfrac{dy}{dx} = 2e^{2x} + 2x \Rightarrow \left(\dfrac{dy}{dx}\right)_{x=0} = 2e^0 + 0 = 2$

Slope of normal $= -\dfrac{1}{2}$, at $x = 0$, $y = 1$

Equation of normal, $y - 1 = \dfrac{1}{2}(x - 0)$

$\Rightarrow x + 2y - 2 = 0$. Its distance from $(0,0) = \dfrac{2}{\sqrt{5}}$

43. (a,c)



The above graph represents that all points inside the triangle formed by the points $(1,3)(5,0)$ and $(-1,2)$ satisfy $3x + 2y \geq 0$, $2x - 3y - 12 \leq 0$.

44. (a, b,c)

T_p of $AP = \dfrac{1}{q(p+q)} = A + (p-1)D$ ...(i)

T_q of $AP = \dfrac{1}{q(p+q)} = A + (q-1)D$ ...(ii)

$T_{p+q} = A + (p+q-1)D.$

and $T_{pq} = A + (pq-1)D.$

Now, solving Eqs. (i) and (ii), we get

$$A = D = \frac{1}{pq(p+q)}$$

$$T_{p+q} = A + (p+q-1)D = (p+q)D = \frac{1}{pq}$$

$$T_{pq} = A + f(p+q-1)D = pqD = \frac{1}{p+q}$$

Sol. 45. (a,b,c,d)

Given $f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3)$

$\Rightarrow f'(x) = 3x^2 + 2x f'(1) + f''(2)$

$\Rightarrow f'(1) = 3 + 2f'(1) + f''(2)$

$\Rightarrow f'(1) + f''(2) = -3$

$\Rightarrow$ and $f''(x) = 6x + 2f'(1)$

$\therefore f''(2) = 12 + 2f'(1)$

$\therefore -2f'(1) + f''(2) = 12$

Solving Eqs. (i) and (ii) we get

$f'(1) = -5$ and $f''(2) = 2$

and $f'''(x) = 6 \Rightarrow f'''(3) = 6$

Substituting the values of $f'(1), f''(2)$ and $f'''(3)$ from Eq. (i), (ii) and (iii) in $f(x)$

$\therefore f(x) = x^3 - 5x^2 + 2x + 6$

$\Rightarrow f(0) = 6, f(1) = 4, f(2) = -2, f(3) = -6$

Hence, $f(0) + f(2) = f(1)$

$f(0) + f(3) = 0$

and $f(1) + f(3) = f(2)$

46. (b,d)

$\because f''(x) < 0$ in $0 \leq x \leq 1$

$\Rightarrow f'(x)$ is decreasing function

Now, $Q'(x) = f'(x) - f'(1-x)$

Case I : If $x \ge (1-x)$

$\therefore \; f'(x) \le f'(1-x)$

$\Rightarrow \; f'(x) - f'(1-x) \le 0$

$\Rightarrow Q'(x) \le 0$

$\therefore Q(x)$ is decreasing, when $x \ge 1-x$ and $0 \le x \le 1$

$\therefore \; x \in \left[\frac{1}{2}, 1\right]$

Case II : If $x \le (1-x)$

$\therefore \; f'(x) \ge f'(1-x)$

$\Rightarrow \; f'(x) - f'(1-x) \ge 0$

$\Rightarrow Q'(x) \ge 0$

$\therefore Q(x)$ is increasing, when $x \le 1-x$ and $0 \le x \le 1$

$\therefore x \in \left[\frac{1}{2}, 1\right]$

47. (a, b,d)

At $(0,0,0), x-y-z-2 = -2 = (-ve)$

at $(2,3,1), x-y-z-2 = 2-3-1-2 = -4(-ve)$

Since, both have same sign $(0,0,0)$ and $(2,3,1)$ line on the same side of the plane.

Distance $= \left|\frac{2-3-1-2}{\sqrt{1^2+1^2+1^2}}\right| = \frac{4}{\sqrt{3}}$

Equation of a line perpendicular to the plane $x-y-z-2=0$ and passing through the point $(2,3,1)$ is

$$\frac{x-2}{1} = \frac{y-3}{-1} = \frac{z-1}{-1} = \lambda$$

A point on the line is $(\lambda+2,\ 3-\lambda,\ 1-\lambda)$ and it lies on the plane $x-y-z-2=0$

if $\lambda+2-3+\lambda-1+\lambda-2=0$

$\Rightarrow \quad \lambda = \frac{4}{3}$

$\therefore$ Foot of perpendicular on the plane is

$$\left(\frac{4}{3}+2, 3-\frac{4}{3}, 1-\frac{4}{3}\right) = \left(\frac{10}{3}, \frac{5}{3}, -\frac{1}{3}\right)$$

So, Image of point P by the plane is $\left(\frac{14}{3}, \frac{1}{3}, \frac{-5}{3}\right)$

48. (2) Let the GP be $a, ar, ar^2, ... (0<r<1)$. From the question, $\frac{a}{1-r} = 3^2 + 3.3 - 9$

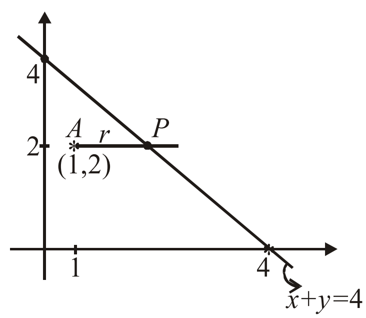
{$\because$ $f'(x) = 3x^2+3+0$; so, $f(x)$ is monotonically increasing; $\therefore f(3)$ is the greatest value in $[-2,3]$.}

Also, $f'(0)=3$. So, $a - ar = 3$.

Solving, $a = 27(1-r)$ and $a(1-r)=3$

we get $r = 2/3$, $4/3$. But $r<1$.

49. (3)



AP $= r = \sqrt{6}/3$

Equation to AP :

$$\frac{x-1}{\cos\theta} = \frac{y-2}{\sin\theta} = r$$

$\therefore \; P \equiv (1 + r\cos\theta, 2 + r\sin\theta)$

'P' will satisfy $x+y=4$

$\therefore \; 1 + \frac{\sqrt{6}}{3}\cos\theta + 2 + \frac{\sqrt{6}}{2}\sin\theta = 4$

$\cos\left(\theta - \frac{\pi}{4}\right) = \frac{\sqrt{3}}{2} = \cos 30°$

$\theta = 75°$ or $15°$

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50. (2)

Here, $d_1 = d\cos(90° - \alpha)$

$d_2 = d\cos(90° - \beta)$

and $d_3 = d\cos(90° - \gamma)$

$\therefore$ $d_1 = d\sin\alpha$

$d_2 = d\sin\beta$

$d_3 = d\sin\gamma$

$\therefore$ $d_1^2 + d_2^2 + d_3^2 = kd^2$

$\Rightarrow d^2\left(\sin^2\alpha + \sin^2\beta + \sin^2\gamma\right) = kd^2 \Rightarrow k = 2$

51. (1)

Ray $x + y = |a|$ in 1st quardant is $x + y = a$.

On solving the equations $x + y = a$ and $ax - y = 1$, we get $y = a - 1$, $x = 1$

Since in 1st quardant $y > 0 \Rightarrow a > 1$

So, value of a_0 is 1.

52. (4)

$$4x\left(\frac{dy}{dx}\right)^2 = y^2 - 4$$

$$4\left[x \cdot 2\frac{dy}{dx}\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right] = 2y\frac{dy}{dx}$$

$$\frac{dy}{dx}\left[4x\frac{d^2y}{dx^2} + \frac{2dy}{dx}\right] = 2$$

53. (2)

Given $x^2 + y^2 = 1$

let $x = \cos\theta$ and $y = \sin\theta$ so $(x + y)^2$ can be written in the form of

$(\cos\theta + \sin\theta)^2 = 2\sin^2\left(\frac{\pi}{4} + \theta\right)$

maximum value = 2 Ans

54. (2)

$f(x) = 1 - x - x^3$ $f'(x) = -1 - 3x^2$ $f(x)$ is decreasing $\forall\ x \in R$.

$f(f(x)) = 1 - f(x) - f^3(x)$

$f(f(x)) > f(1 - 5x)$

$f(x)$ is decreasing hence.

$f(x) < 1 - 5x.$

$1 - x - x^3 < 1 - 5x$

$x(x^2 - 4) > 0$

$\Rightarrow$ $x \in (-2, 0) \cup (2, \infty)$.

55. (4)

We have the points $(1,-2)$ and $(3,4)$ are equidistance from the line $tx - y - 1 = 0$

$\Rightarrow t + 2 - 1 = \pm(3t - 4 - 1)$

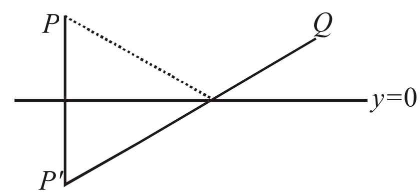
$\Rightarrow t + 1 = \pm(3t - 5)$ $\Rightarrow t = 1, 3$

So, sum of possible values of t is 4.

56. (A)$\rightarrow$r; (B)$\rightarrow$s; (C)$\rightarrow$r; (D)$\rightarrow$p,q,r

(A) The image of P in the x-axis is $P'(1,-1)$.

Equation of $P'Q$ is $y + 1 = \frac{3}{3}(x - 1)$, which intersects x-axis at $(2,0)$



(B) Value of $a_1a_2 + b_1b_2 + c_1c_2$ for lines $x + 2y + 4 = 0$ and $-4x - 2y + 1 = 0$ is negative then acute angle bisector,

$$\frac{x + 2y + 4}{\sqrt{5}} = +\frac{(-4x - 2y + 1)}{\sqrt{20}}$$

$\Rightarrow 2x + 4y + 8 = -4x - 2y + 1$

$\Rightarrow 6x + 6y + 7 = 0$ $\Rightarrow m + 3 = 7$

(C) As two of the lines are parallel, or all lines concurrent, they do not form a triangle.

(D) The line $x = y$ cuts the lines $x + y = \pm 6$ at $(-3,-3)$ and $(3,3) \Rightarrow -3 < a < 3.$

$\therefore [|a|] = 0, 1, 2$

57. (A)$\rightarrow$s, (B)$\rightarrow$r, (C)$\rightarrow$q, (D)$\rightarrow$q

(A) $f(x + y) = f(x) \cdot f(y) \Rightarrow f(0) = 1$

Now,

$$f'(5)=\lim_{h\to 0}\frac{f(5+h)-f(5)}{h}=\lim_{h\to 0}\frac{f(5)\{f(h)-1\}}{h}$$

$$=2\lim_{h\to 0}\frac{f(h)-f(0)}{h}=2f'(0)=2(3)=6$$

(B) Slope of normal $=\tan 135°=-1$

$\Rightarrow \frac{-1}{f'(3)}=-1 \quad \Rightarrow f'(3)=-1$

(C) $F'(x)=x^2\int_1^x f'(u)\,du\,(1)$

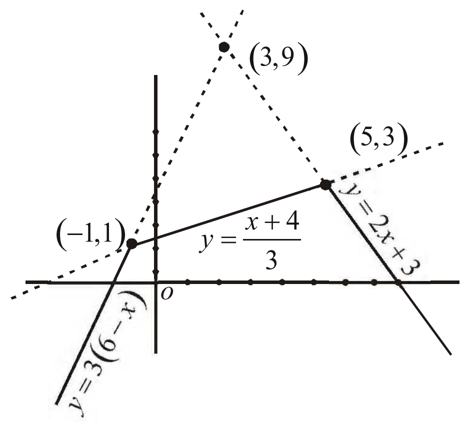
$\Rightarrow F''(1)=0$

Also, $f''(x)=x^2 f(x)\cdot 1-\int_1^x f'(u)\,du\,(2x)$

$\Rightarrow F''(1)=f(1)=3$

So, $F'(1)+F''(1)=3.$

(D)



from the graph, It is clear that maximum value of $f(x)=\min\left\{2x+3,\frac{x+4}{3},3(6-x)\right\}$ is 3.

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