SOLUTIONS OF IIT-JEE 2010

Questions are given in short here and answer corresponding to each question is mentioned. So Answers for each paper set is covered. Detailed solutions are available at our center and website www. momentumacademy.com

PAPER - I

[MATHEMATICS]

QUESTIONS WITH ONLY ONE CORRECT ANSWERS

29. Equation of the plane containing the straight line $\frac{x}{2}=\frac{y}{3}=\frac{z}{4}$ and perpendicular to the plane containing the straight line $\frac{x}{3}=\frac{y}{4}=\frac{z}{2}$ and $\frac{x}{4}=\frac{y}{2}=\frac{z}{3}$ is

(a) $x+2y-2z=0$ (b) $3x+2y-2z=0$

(c) $x-2y+z=0$ (d) $5x+2y-4z=0$

Sol.29.(c)

DR of Normal to plane containing given lines

$$=(3,4,2)\times(4,2,3)=(8,-1,-10)$$

let normal of required plane be (a,b,c)

Given

$$(a,b,c)\perp(2,3,4) \quad \Rightarrow 2a+3b+4c=0$$
$$(a,b,c)\perp(8,-1,-10) \Rightarrow 8a-b-10c=0$$
$$\frac{a}{-26}=\frac{b}{52}=\frac{c}{-26}$$

$\therefore \ (a,b,c)=(1,-2,1)$

Ans. (c) $x-2y+z=0$

30. If the angles A, B and C of a triangle are in an arithmetic progression and if a, b and c denote the lengths of the sides opposite to A, B and C respectively, then the value of the expression $\frac{a}{c}\sin 2C+\frac{c}{a}\sin 2A$ is

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$

(c) 1 (d) $\sqrt{3}$

Sol.30.(d)

$$\angle A+\angle B+\angle C=180^0$$

Given $2\angle B=\angle A+\angle C$

$$\Rightarrow \angle B=60^0$$

$$\frac{a}{c}\sin 2C+\frac{c}{a}\sin 2A$$

$$=\frac{a}{c}[2\sin C\cos C]+\frac{c}{a}[2\sin A\cos A]$$

$$=2\left[\frac{\sin C}{c}a\cos C+\frac{\sin A}{a}c\cos A\right]$$

$$=2\frac{\sin B}{b}[a\cos C+c\cos A]$$

$$=2\frac{\sqrt{3}}{2}\left[\frac{a\cos C+c\cos A}{b}\right]=\sqrt{3}\cdot\frac{b}{b}=\sqrt{3}$$

31. Let f, g and h be real-valued functions defined on the interval $[0,1]$ by $f(x)=e^{x^2}+e^{-x^2}$, $g(x)=xe^{x^2}+e^{-x^2}$ and $h(x)=x^2e^{x^2}+e^{-x^2}$. If a,b and c denote, respectively, the absolte maximum of f,g and h on $[0,1]$, then

(a) $a=b$ and $c\neq b$ (b) $a=c$ and $a\neq b$

(c) $a\neq b$ and $c\neq b$ (d) $a=b=c$

Sol.31.(d)

$$f(x)=e^{x^2}+e^{-x^2}$$

$$f'(x)=2x\left[e^{x^2}-e^{-x^2}\right]$$

$$f'(x)=0\Rightarrow x=0\ f'(x)\geq 0\ \forall x\in[0,1]$$

$$g(x)=xe^{x^2}+e^{-x^2}$$

$$g'(x)=e^{x^2}+x\left(2xe^{x^2}\right)-2xe^{-x^2}$$

$$=e^{x^2}+2x\left[xe^{x^2}-e^{-x^2}\right]$$

$$\left(g'(x)>0\ \forall x\in[0,1]\right)$$

$$h(x)=x^2e^{x^2}+e^{-x^2}$$

$$h'(x)=2xe^{x^2}+x^2 2xe^{x^2}-2xe^{-x^2}$$

$$=2xe^{x^2}\left(x^2+1\right)-2xe^{-x^2}$$

$$h'(x)=0\Rightarrow x=0$$

$$h'(x)\geq 0\ \forall x\in[0,1]$$

Ans. $a=b=c$

32. Let p and q be real number such that $p\neq 0, p^3\neq q$ and $p^3\neq -q$. If α and β are nonzero complex numbers satisfying $\alpha+\beta=-p$ and $\alpha^3+\beta^3=q$, then quadratic equation having $\dfrac{\alpha}{\beta}$ and $\dfrac{\beta}{\alpha}$ as its roots is

(a) $\left(p^3+q\right)x^2-\left(p^3+2q\right)x+\left(p^3+q\right)=0$

(b) $\left(p^3+q\right)x^2-\left(p^3-2q\right)x+\left(p^3+q\right)=0$

(c) $\left(p^3-q\right)x^2-\left(5p^3-2q\right)x+\left(p^3-q\right)=0$

(d) $\left(p^3-q\right)x^2-\left(5p^3+2q\right)x+\left(p^3-q\right)=0$

Sol.32.(b)

Sum of roots $=\dfrac{\alpha}{\beta}+\dfrac{\beta}{\alpha}=\dfrac{\alpha^2+\beta^2}{\alpha\beta}$

product of roots $=\dfrac{\alpha}{\beta}\cdot\dfrac{\beta}{\alpha}=1$

$\therefore$ equation will be $x^2-\left(\dfrac{\alpha^2+\beta^2}{\alpha\beta}\right)x+1=0$

given $\alpha+\beta=-p$ ...(1)

$\alpha^3+\beta^3=q$ ....(2)

cubing (1) on both sides

$$\alpha^3+\beta^3+3\alpha\beta(\alpha+\beta)=(-p)^3$$

$$\Rightarrow q+3\alpha\beta(-p)=-p^3$$

$$\Rightarrow \alpha\beta=\frac{-p^3-q}{-3p}=\frac{p^3+q}{3p} \quad ....(3)$$

squarring (1)

$$\alpha^2+\beta^2+2\alpha\beta=(-p)^2$$

$$\alpha^2+\beta^2+2\cdot\left(\frac{p^3+q}{3p}\right)=p^2$$

$$\Rightarrow \alpha^2+\beta^2=p^2-2\left(\frac{p^3+q}{3p}\right)$$

$$=\frac{p^3-2q}{3p} \quad ....(4)$$

putting these values in required equation we get

$$\left(p^3+q\right)x^2-\left(p^3-2q\right)x+\left(p^3+q\right)=0$$

33. The value of $\lim\limits_{x\to 0}\dfrac{1}{x^3}\displaystyle\int_0^x\dfrac{t\ \ell n(1+t)}{t^4+4}dt$ is

(a) 0 (b) $\dfrac{1}{12}$

(c) $\dfrac{1}{24}$ (d) $\dfrac{1}{64}$

Sol.33.(b)

$$\lim_{x\to 0}\frac{1}{x^3}\int_0^x\frac{t\ln(1+t)}{t^4+4}dt$$

$$= \lim_{x\to 0} \frac{\int_0^x \frac{t\ln(1+t)}{t^4+4}}{x^3}$$

Applying L' Hospital Rule

$$\lim_{x\to 0} \frac{\left(x\ln(1+x)/x^4+4\right)-(0)}{3x^2}$$

$$= \lim_{x\to 0} \frac{x\ln(1+x)}{(x^4+4)3x^2} = \lim_{x\to 0} \frac{\ln(1+x)}{(x^4+1)(3x)}$$

Again L' Hospital

$$= \lim_{x\to 0} \frac{1}{(1+x)\left[3(x^4+4)+4x^3 3x\right]}$$

$$= \frac{1}{(1)[12]} = \frac{1}{12}$$ Ans.

34. Then number of 3×3 matrices A whose entries are either 0 or 1 and for which the system $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ has exactly two distinct solutions, is

(a) 0 (b) $2^9 - 1$

(c) 168 (d) 2

Sol.34.(a)

$A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ gives 3 linear equation in x, y, z which represents equations of 3 planes. Now, 3 planes cannot meet at exactly 2 distinct points. Hence there is no such matrix possible.

35. Let P, Q, R and S be the points on the plane with position vectors $-2\hat{i}-\hat{j}, 4\hat{i}, 3\hat{i}+3\hat{j}$ and $-3\hat{i}+2\hat{j}$ respectively. The quadrilateral $PQRS$ must be a

(a) Parallelogram, which is neither a rhombus nor a rectangle

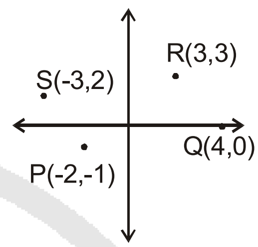
(b) square

(c) rectangle, but not a square

(d) rhombus, but not a square

Sol.35.(a)

$P = -2i - j, Q = 4i, R = 3i + 3j, S = -3i + 2j$



$PQ \,\&\, PS$ are not perpendicular or equal in length but Diagonals PR and SQ bisect.

$\therefore PQRS$ is a parallelogram but not rhombus or rectangle

36. Let ω be a complex cube root of unity with $\omega \neq 1$. A fair die is thrown three times. If r_1, r_2 and r_3 are the number obtained on the die, then the probability that $\omega^{r_1} + \omega^{r_2} + \omega^{r_3} = 0$ is

(a) $\frac{1}{18}$ (b) $\frac{1}{9}$

(c) $\frac{2}{9}$ (d) $\frac{1}{36}$

Sol.36.(c)

$\omega^{r_1} + \omega^{r_2} + \omega^{r_3} = 0$

out of r_1, r_2, r_3 one must be 3 or 6

out of r_1, r_2, r_3 one must be 2 or 5

out of r_1, r_2, r_3 one must be 1 or 4

Favourable No. of ways $= {}^3C_1 \times 2 \times {}^2C_1 \times 2 \times {}^1C_1 \times 2 = 48$

Total cases = 216

Required probability $= \frac{48}{216} = \frac{2}{9}$

QUESTIONS WITH MULTIPLE CORRECT CHOICE TYPE

37. The value(s) of $\int_0^1 \frac{x^4(1-x)^4}{1+x^2}dx$ is (are)

(a) $\frac{22}{7}-\pi$ (b) $\frac{2}{105}$

(c) 0 (d) $\frac{71}{15}-\frac{3\pi}{2}$

Sol.37.(a)

$$\int_0^1 \frac{x^4(1-x)^4}{1+x^2}dx$$

$$=\int_0^1 \frac{(x^4-1+1)(1-x)^4}{1+x^2}dx$$

$$=\int_0^1 \frac{(x^4-1)(1-x)^4}{1+x^2}dx+\int_0^1 \frac{(1-x)^4}{1+x^2}dx$$

$$=\int_0^1 (x^2-1)(1-x)^4\,dx+\int_0^{\pi/4}(1-\tan\theta)^4\,d\theta$$

(on putting $x=\tan\theta$)

$$=\int_0^1\left((1-x)^2-1\right)\left(1-(1-x)\right)^4dx+$$

$$\int_0^{\pi/4}\left(\sec^2\theta-2\tan\theta\right)^2d\theta$$

$$=\int_0^1(x^2-2x)(x^4)\,dx+\int_0^{\pi/4}\sec^4\theta\,d\theta+$$

$$\int_0^{\pi/4}4\tan^2\theta\,d\theta-\int_0^{\pi/4}4\tan\theta\sec^2\theta\,d\theta$$

$$=\frac{1}{7}-\frac{2}{6}+\frac{4}{3}+4\left(1-\frac{\pi}{4}\right)-2$$

$$=\frac{1}{7}+\frac{1}{3}+4-\pi-\frac{1}{2}=\frac{22}{7}-\pi$$

38. Let z_1 and z_2 be two distinct complex numbers and let $z=(1-t)z_1+tz_2$ for some real number t with $0<t<1$. If $Arg(w)$ denotes the principal argument of a nonzero complex number w, then

(a) $|z-z_1|+|z-z_2|=|z_1-z_2|$

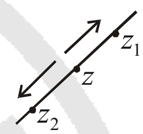
(b) $Arg(z-z_1)=Arg(z-z_2)$

(c) $\begin{vmatrix} z-z_1 & \bar{z}-\bar{z}_1 \\ z_2-z_1 & \bar{z}_2-\bar{z}_1 \end{vmatrix}=0$

(d) $Arg(z-z_1)=Arg(z_2-z_1)$

Sol.38.(a)(c)(d)

z lies on the line joining z_1 & z_2 on the portion between them.



(Distance between z_1 & z) +

(Distance between z & z_2) =distance between z_1 & z_2 and direction of z w.r.t. z_1 & z_2 w.r.t. z_1 are same.

39. Let f be a real - valued function defined on the interval $(0,\infty)$ by

$f(x)=\ell n\,x+\int_0^x\sqrt{1+\sin t}\,dt$. Then which of the following statement(s) is (are) true ?

(a) $f''(x)$ exists for all $x\in(0,\infty)$

(b) $f'(x)$ exists for all $x\in(0,\infty)$ and f' is continuous on $(0,\infty)$, but not

(c) there exists $\alpha>1$ such that $|f'(x)|<|f(x)|$ for all $x\in(\alpha,\infty)$

(d) there exists $\beta>0$ such that $|f(x)|+|f'(x)|\le\beta$ for all $x\in(0,\infty)$

Sol.39.(b)(c)

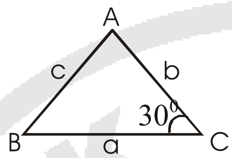
40. Let ABC be a triangle such that $\angle ACB=\frac{\pi}{6}$ and let a,b and c denote the lengths of the sides opposite to A, B and C respectively. The value(s) of x for which $a = x^2 + x + 1$, $b = x^2 - 1$ and $c = 2x + 1$ is (are)

(a) $-\left(2+\sqrt{3}\right)$ (b) $1+\sqrt{3}$

(c) $2+\sqrt{3}$ (d) $4\sqrt{3}$

Sol.40.(b)

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{\left(x^2+x+1\right)^2 + \left(x^2-1\right)^2 - \left(2x+1\right)^2}{2\left(x^2+x+1\right)\left(x^2-1\right)}$$

$$\Rightarrow \sqrt{3} = \frac{\left(x^2+x+1\right)^2 + \left[\left(x^2+2x\right)\left(x^2-2x-2\right)\right]}{\left(x^2+x+1\right)\left(x^2-1\right)}$$

$$\Rightarrow \sqrt{3} = \frac{\left(x^2+x+1\right)^2 + x\left(x+2\right)\left(x-2\right)\left(x+1\right)}{\left(x^2+x+1\right)\left(x^2-1\right)}$$

$$\Rightarrow \sqrt{3} = \frac{x^2+x+1}{\left(x^2-1\right)} + \frac{x\left(x^2-4\right)}{\left(x^2+x+1\right)\left(x-1\right)}$$

$$\Rightarrow x = 1+\sqrt{3}$$

41. Let A and B be two distinct points on the parabola $y^2 = 4x$. If the axis of the parabola touches a circle of radius r having AB as its diameter, then the slope of the line joining A and B can be

(a) $-\frac{1}{r}$ (b) $\frac{1}{r}$

(c) $\frac{2}{r}$ (d) $\frac{-2}{r}$

Sol.41.(c)(d)

Let A and B on parabola be

$\left(t_1^2, 2t_1\right)$ and $\left(t_2^2, 2t_2\right)$ respectively

Slope of $AB = \frac{2\left(t_2-t_1\right)}{t_2^2-t_1^2} \Rightarrow m = \frac{2}{t_2+t_1}$ ...(1)

If circle touches x-axis and its centre is (α, β)

then $|\beta| = r$

$$\left|\left(\frac{2t_1+2t_2}{2}\right)\right| = r$$

$$\Rightarrow \left|\frac{2}{m}\right| = r \qquad \text{from (1)}$$

$$\Rightarrow m = \pm\frac{2}{r}$$

QUESTIONS WITH COMPREHENSION TYPE
PARAGRAPH 42 TO 43

The circle $x^2 + y^2 - 8x = 0$ and hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ intersect at the points A and B.

42. Equation of a common tangent with positive slope to the circle as well as to the hyperbola is

(a) $2x - \sqrt{5}y - 20 = 0$

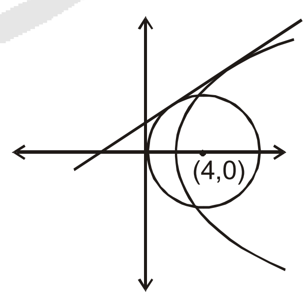
(b) $2x - \sqrt{5}y + 4 = 0$

(c) $3x - 4y + 8 = 0$

(d) $4x - 3y + 4 = 0$

Sol.42.(b)

The tangent of hyperbola can be taken



$$y = mx + \sqrt{a^2m^2 - b^2}$$

$$\Rightarrow y = mx + \sqrt{9m^2 - 4} \qquad .....(1)$$

for this to be tangent to given circle its distance from centre = radius of circle

$$\left|\frac{m\times 4-0+\sqrt{9m^2-4}}{\sqrt{m^2+1}}\right|=4$$

$$\Rightarrow 4m+\sqrt{9m^2-4}=4\sqrt{m^2+1}$$

solving by squarring & simplifying gives

$$m=\frac{2}{\sqrt{5}}$$

Ans. (B). $2x-\sqrt{5}y+4=0$

43. Equation of the circle with AB as its diameter is

(a) $x^2+y^2-12x+24=0$

(b) $x^2+y^2+12x+24=0$

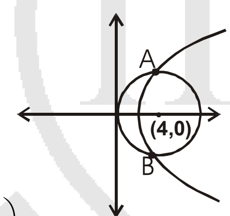
(c) $x^2+y^2+24x-12=0$

(d) $x^2+y^2-24x-12=0$

Sol.43.(a)

$$x^2+y^2-8x=0$$

$$\frac{x^2}{9}-\frac{y^2}{4}=1$$



solving $x^2+4\left(\frac{x^2}{9}-1\right)-8x=0$ ....(1)

From option, points A, B can have x-cordinates $6,-6,12,-12$ out of which only 6 satisfies equation (1)

Ans. (A) equation of circle

$$x^2+y^2-12x+24=0$$

PARAGRAPH 44 TO 46

Let p be an odd prime number and T_p be the following set of 2×2 matrices :

$$T_p=\left\{A=\begin{bmatrix} a & b \\ c & a\end{bmatrix}: a, b, c \in \{0, 1, 2, ..., p-1\}\right\}$$

44. The number of A in T_p such that A is either symmetric or skew-symmetric or both, and det (A) divisible by p is

(a) $(p-1)^2$ (b) $2(p-1)$

(c) $(p-1)^2+1$ (d) $2p-1$

Sol.44.(d)

45. The number of A in T_p such that the trace of A is not divisible by p but det (A) is divisible by p is

[**Note :** The trace of a matrix is the sum of its diagonal entries.]

(a) $(p-1)(p^2-p+1)$

(b) $p^3-(p-1)^2$

(c) $(p-1)^2$

(d) $(p-1)(p^2-2)$

Sol.45.(c)

46. The number of A in T_p such that det (A) is not divisible by p is

(a) $2p^2$ (b) p^3-5p

(c) p^3-3p (d) p^3-p^2

Sol.46.(d)

QUESTIONS WITH INTEGER TYPE

47. The line $2x+y=1$ is tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. If this line passes through the point of intersectin of the nearest directrix and the x-axis, then the eccentricity of the hyperbola is

Sol.47.(2)

48. If the distance between the plane $Ax-2y+z=d$ and the plane containing the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ is $\sqrt{6}$, then $|d|$ is

Sol.48.(6)

$(A,-2,1)=DR$ of given plane

for DR of 2nd plane

$(2,3,4)\times(3,4,5)\Rightarrow(-1,2,-1)$

$\because$ planes are parallel

$$\frac{A}{-1}=\frac{-2}{2}=\frac{1}{-1}\Rightarrow A=1$$

Now distance between planes =distance of first plae from a point (1,2,3) on given line.

$$\left|\frac{1\times1-2\times2+3-d}{\sqrt{1^2+2^2+1^2}}\right|=\sqrt{6}$$

$$|d|=\sqrt{6}\times\sqrt{6}=6$$

Ans.

49. For any real number x, let $[x]$ denote the largest integer less than or equal to x. Let f be a real valued function defined on the interval $[-10, 10]$ by

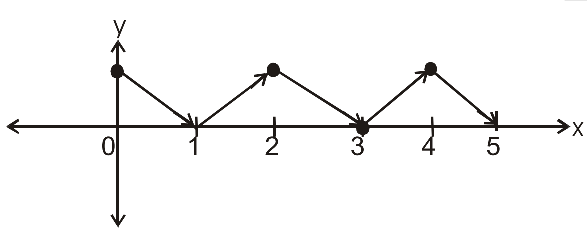
$$f(x)=\begin{cases} x-[x] & if\ [x]\ is\ odd \\ 1+[x]-x & if\ [x]\ is\ even \end{cases}$$

Then the value of $\frac{\pi^2}{10}\int_{-10}^{10} f(x)\cos\pi x\,dx$ is

Sol.49.(4)

$$f(x)=\begin{cases} x-[x], \text{if } [x] \text{is odd} \\ 1+[x]-x, \text{if } [x] \text{ is even} \end{cases}$$

$$f(x)=\begin{cases} \{x\}, \text{if } [x] \text{is odd} \\ 1-\{x\}, \text{if } [x] \text{ is even} \end{cases}$$



graph of $f(x)$

$$\frac{\pi^2}{10}\int_{-10}^{10} f(x)\cos\pi x\,dx$$

$\because f(x)$ is even with period $='2'$

$$\Rightarrow\frac{2\pi^2}{10}\int_0^{10} f(x)\cos\pi x\,dx$$

$\cos\pi x$ is also periodic with period $='2'$

$$\Rightarrow\frac{2\pi^2}{10}(5)\int_0^2 f(x)\cos\pi x\,dx$$

$$\Rightarrow\pi^2\left[\int_0^1(1-\{x\})\cos\pi x\,dx+\int_1^2\{x\}\cos\pi x\,dx\right]$$

$$\Rightarrow\pi^2\left[\int_0^1(1-x)\cos\pi x\,dx+\int_1^2 x\cos\pi x\,dx\right]$$

$$\Rightarrow\pi^2\left[\int_0^1\cos\pi x\,dx-\int_0^1 x\cos\pi x\,dx+\int_1^2 x\cos\pi x\,dx\right]$$

$$\Rightarrow\pi^2\left[0-\left(-\frac{1}{\pi^2}-\frac{1}{\pi^2}\right)+\left(\frac{1}{\pi^2}+\frac{1}{\pi^2}\right)\right]$$

$$=\pi^2\left[\frac{2}{\pi^2}+\frac{2}{\pi^2}\right]=4$$

50. Let ω be the complex number $\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}$. Then the number of distinct complex numbers z satisfying $\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix}=0$ is equal to

Sol.50.(1)

$R_1\to R_1+R_2+R_3$ using $1+\omega+\omega^2=0$ and taking z common from R_1

$$\Rightarrow z\begin{vmatrix} 1 & 1 & 1 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix}=0$$

$\Rightarrow$ Now $C_2\to C_2-C_1, C_3\to C_3-C_1$

$$z\begin{vmatrix} 1 & 0 & 0 \\ \omega & z+\omega^2-\omega & 1-\omega \\ \omega^2 & 1-\omega^2 & z+\omega-\omega^2 \end{vmatrix}=0$$

Expanding we get

$$z\left[\left(z+\omega^2-\omega\right)\left(z+\omega-\omega^2\right)-\left(1-\omega\right)\left(1-\omega^2\right)\right]=0$$

$$z\left(\left(z-\left(\omega-\omega^2\right)\right)\left(z+\left(\omega-\omega^2\right)\right)-3\right)=0$$

$$\Rightarrow z\left(z^2-\left(\omega-\omega^2\right)^2-3\right)=0$$

$$\Rightarrow z\left(z^2-\left(\omega^2+\omega\right)-1\right)=0 \Rightarrow z^3=0$$

51. Let $S_k, k=1,2,.....,100$, denote the sum of the infinite geometric series whose first term is $\frac{k-1}{k!}$ and the common ratio is $\frac{1}{k}$. Then the value of $\frac{100^2}{100!}+\sum_{k=1}^{100}\left|\left(k^2-3k+1\right)S_k\right|$ is

Sol.51.(3) $a=\frac{k-1}{k!}, r=\frac{1}{k}, S_k=\frac{1}{(k-1)!}$

$$\frac{100^2}{100!}+\sum_{k=1}^{100}\left|\left(k^2-3k+1\right)S_k\right|$$

$$=\frac{100^2}{100!}+\sum_{k=1}^{100}\left|\frac{(k-1)^2-k}{(k-1)!}\right|$$

$$=\frac{100^2}{100!}+\sum_{k=1}^{100}\left|\frac{k-1}{(k-2)!}-\frac{k}{(k-1)!}\right|$$

$$=\frac{100^2}{100!}+\left|\frac{1}{0!}-\frac{2}{1!}\right|+\left|\frac{2}{1!}-\frac{3}{2!}\right|+\left|\frac{3}{2!}-\frac{4}{3!}\right|.............+\left|\frac{99}{98!}-\frac{100}{99!}\right|$$

$$=\frac{100^2}{100!}+\left(\frac{2}{1!}-\frac{1}{0!}\right)+\left(\frac{2}{1!}-\frac{3}{2!}\right)+\left(\frac{3}{2!}-\frac{4}{3!}\right)............+\left(\frac{99}{98!}-\frac{100}{99!}\right)$$

= 3

52. The number of all possible values of θ, where $0<\theta<\pi$ for which the system of equations

$$\left(y+z\right)\cos 3\theta=\left(xyz\right)\sin 3\theta$$

$$x\sin 3\theta=\frac{2\cos 3\theta}{y}+\frac{2\sin 3\theta}{z}$$

$$\left(xyz\right)\sin 3\theta=\left(y+2z\right)\cos 3\theta+(\sin 3\theta)y$$

have a solution $\left(x_0, y_0, z_0\right)$ with $y_0z_0\neq 0$ is

Sol.52.(3)

Given, $\left(\frac{1}{xz}+\frac{1}{xy}\right)\cdot\cos 3\theta=\sin 3\theta$,

$\sin 3\theta=\frac{1}{xy}\cdot 2\cos 3\theta+2\cdot\sin 3\theta\cdot\frac{1}{xz}$ and

$$\sin 3\theta=\left(\frac{1}{xz}+\frac{2}{xy}\right)\cos 3\theta+\frac{1}{xz}\sin 3\theta$$

$$\frac{1}{xy}=p, \frac{1}{yz}=q, \frac{1}{zx}=r$$

$\Rightarrow\left(r+p\right)\cos 3\theta=\sin 3\theta$,

$\sin 3\theta=p\left(2\cos 3\theta\right)+2r\sin 3\theta$ and

$\sin 3\theta=\left(r+2p\right)\cos 3\theta+r\sin 3\theta$

$\Rightarrow p\cos 3\theta+0\cdot q+r\cos 3\theta=\sin 3\theta$ ...(1),

$2p\cdot\cos 3\theta+0\cdot q+2r\sin 3\theta=\sin 3\theta$ ..(2)

and

$2p\cdot\cos 3\theta+0\cdot q+r\cos 3\theta=\sin 3\theta$ ..(3)

From equation 1 & 2

$p\cos 3\theta+r\sin 3\theta=0\Rightarrow\tan 3\theta=\frac{-p}{r}$ ....(4)

From equation 2 & 3

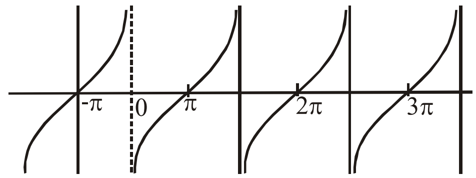
$r\cos 3\theta=0$ ....(5)

From equation 3 & 1 $p\cos 3\theta=0$ ....(6)

$\therefore$ from 5 & 6 $r=p$

From equation 4 $\tan 3\theta=-1$

So 3 values are possible



53. Let f be a real-valued differentiable function on R (the set of all real numbers) such that $f(1)=1$. If the y intercept of the tangent at any point $P(x,y)$ on the curve $y=f(x)$ is equal to the cube of the abscissa of P, then the value of $f(-3)$ is equal to

Sol.53.(9)

Tangent at (x,y)

$(Y-y)=f'(x)(X-x)$

y -intercept of tangent $= y - x f'(x)$

Given $y - x\dfrac{dy}{dx} = x^3$

$\Rightarrow \dfrac{dy}{dx} - \dfrac{y}{x} = -x^2$

solution is , $\dfrac{y}{x} = \int \dfrac{-x^2}{x} dx + C$

$\Rightarrow y = -\dfrac{x^3}{2} + Cx$

Given $f(1)=1$

$\Rightarrow C = 3/2$

$\therefore\ y = \dfrac{-x^3}{2} + \dfrac{3}{2}x$

So, $f(-3) = \dfrac{27}{2} + \left(\dfrac{-9}{2}\right) = \dfrac{16}{2} = 9$

54. The number of values of θ in the interval $\left(-\dfrac{\pi}{2},\dfrac{\pi}{2}\right)$ such that $\theta \neq \dfrac{n\pi}{5}$ for $n=0$, ±1, ±2 and $\tan\theta = \cot 5\theta$ as well as $\sin 2\theta = \cos 4\theta$ is

Sol.54.(3)

$\tan\theta = \cot 5\theta$

$\Rightarrow \cos\theta\cdot\cos 5\theta - \sin\theta\cdot\sin 5\theta = 0$

$\Rightarrow \cos 6\theta = 0$

$\Rightarrow 6\theta = (2n+1)\dfrac{\pi}{2} \Rightarrow \theta = (2n+1)\dfrac{\pi}{12}$ ...(1)

Now, $\sin 2\theta = \cos 4\theta$

$\sin 2\theta = 1 - 2\sin^2 2\theta$

$2\sin^2 2\theta + \sin 2\theta - 1 = 0 \Rightarrow \sin 2\theta = -1, 1/2$

$2\theta = (4n-1)\pi/2, n\pi + (-1)^n \pi/6$

$\theta = (4n-1)\pi/4, \dfrac{n\pi}{2} + (-1)^n \dfrac{\pi}{12}$ ....(2)

From (1) & (2), common values

$\theta = 15^0, 75^0, -45^0$

Ans. 3 values

55. The maximum value of the expression $\dfrac{1}{\sin^2\theta + 3\sin\theta\cos\theta + 5\cos^2\theta}$ is

Sol.55.(2)

$$\frac{1}{\sin^2\theta + 3\sin\theta\cos\theta + 5\cos^2\theta}$$

$$= \frac{1}{\dfrac{1-\cos 2\theta}{2} + \dfrac{3\sin 2\theta}{2} + \dfrac{5(1+\cos 2\theta)}{2}}$$

$$= \frac{2}{1-\cos 2\theta + 3\sin 2\theta + 5(1+\cos 2\theta)}$$

$$= \frac{2}{4\cos 2\theta + 3\sin 2\theta + 6}$$

For maximum value

$4\cos 2\theta + 3\sin 2\theta + 6$ should be least

$\because 6-5 \le 4\cos 2\theta + 3\sin 2\theta + 6 \le 5+6$

$\therefore$ Maximum value $= \dfrac{2}{1} = 2$

56. If $\vec{a}$ and $\vec{b}$ are vectors in space given by $\vec{a} = \dfrac{\hat{i} - 2\hat{j}}{\sqrt{5}}$ and $\vec{b} = \dfrac{2\hat{i} + \hat{j} + 3\hat{k}}{\sqrt{14}}$, then the value of $\left(2\vec{a} + \vec{b}\right) \cdot \left[\left(\vec{a} \times \vec{b}\right) \times \left(\vec{a} - 2\vec{b}\right)\right]$ is.

Sol.56.(5)

$$\left[2\overline{a} + \overline{b} \quad \overline{a} \times \overline{b} \quad \overline{a} - 2\overline{b}\right]$$

$$= \left[2\overline{a} \quad \overline{a} \times \overline{b} \quad -2\overline{b}\right] + \left[\overline{b} \quad \overline{a} \times \overline{b} \quad \overline{a}\ \right]$$

$$= -4\left[\overline{a} \quad \overline{a} \times \overline{b} \quad \overline{b}\right] - \left[\overline{a} \quad \overline{a} \times \overline{b} \quad \overline{b}\ \right]$$

$$= -5\left(\overline{a} \times \left(\overline{a} \times \overline{b}\right) \cdot \overline{b}\right)$$

$$= -5\left(\left(\overline{a} \cdot \overline{b}\right)^2 - \left(\overline{a} \cdot \overline{a}\right)\left(\overline{b} \cdot \overline{b}\right)\right)$$

Now $\overline{a} = \left(\dfrac{i - 2j}{\sqrt{5}}\right), \overline{b} = \dfrac{2i + j + 3k}{\sqrt{14}}$

$\overline{a} \cdot \overline{a} = 1 = \overline{b} \cdot \overline{b}$ & $\overline{a} \cdot \overline{b} = 0$

$$\therefore -5\left(\left(\overline{a} \cdot \overline{b}\right)^2 - \left(\overline{a} \cdot \overline{a}\right)\left(\overline{b} \cdot \overline{b}\right)\right) = 5$$

$$= -5\left(\left(\overline{a} \cdot \overline{b}\right)^2 - \left(\overline{a} \cdot \overline{a}\right)\left(\overline{b} \cdot \overline{b}\right)\right)$$

Now $\overline{a} = \left(\dfrac{i - 2j}{\sqrt{5}}\right), \overline{b} = \dfrac{2i + j + 3k}{\sqrt{14}}$

$\overline{a} \cdot \overline{a} = 1 = \overline{b} \cdot \overline{b}$ & $\overline{a} \cdot \overline{b} = 0$

$$\therefore -5\left(\left(\overline{a} \cdot \overline{b}\right)^2 - \left(\overline{a} \cdot \overline{a}\right)\left(\overline{b} \cdot \overline{b}\right)\right) = 5$$