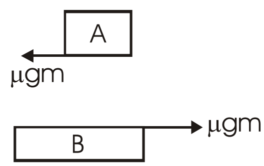
1.(c) Force acting between A and B will be μmg.

Retardation of A is $a_A = \frac{\mu mg}{m} = \mu g$.

and acceleration of B is $a_B = \frac{\mu mg}{2m} = \frac{\mu g}{2}$



Accelration of B relative to A is

$$a_{BA} = a_A + a_B = \frac{3\mu g}{2}$$

Substituting $\mu = \frac{1}{2}$

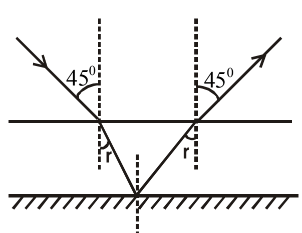
$$a_{BA} = \frac{3g}{4}$$

2.(c) $P = FV$ or $P = ma \cdot V = m\left(\frac{VdV}{ds}\right)\cdot V$

$$\int_{V_1}^{V_2} V^2 dV = \frac{P}{m}\int_0^s ds, \quad \frac{1}{3}\left(V_2^3 - V_1^3\right) = \frac{P}{m}\times s$$

$$S = \frac{m}{3P}\left(V_2^3 - V_1^3\right)$$

3.(a) From ray diagram it is clear that angle between incident ray and emergent ray is 90^0.



4.(c)

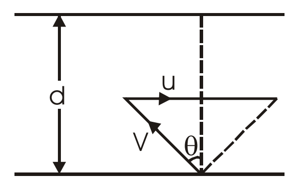
5.(a,c)

(c) At starting point spring force is zero . The only force is mg(downwards). Hence max acceleration downward is g.

(d) When spring is detached only gravity is left reterdation has reduced There force block rise to more height.

6. (a) and (c)

Time required for crossing, $t = \frac{d}{V\cos\theta}$



If he cross the river in minimum time, $\cos\theta = 1$

$$t_{\min} = \frac{d}{V} \Rightarrow x = \frac{du}{V}$$

$$x = \left(u - V\sin g\theta\right)t = \frac{ud}{V}\sec\theta - d\tan\theta$$

For x to be minimum, $\frac{dx}{d\theta} = 0$

This gives $\sin\theta = \frac{V}{u}$

7. (a, c)

Let us go over to the reference frame connected with the platform. initial speed of body is $-V_0$ and the final is zero. We have

$$a = \frac{T}{m} = \frac{\mu mg}{m} = \mu g$$

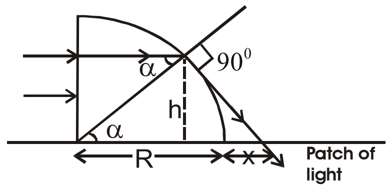
Time, $t = \frac{0-\left(-V_0\right)}{a}$

$$= \frac{V_0}{\mu g} = \frac{4}{0.2\times 10} = 2 \text{ seconds}$$

$$x = -V_0 t + \frac{at^2}{2} = \frac{V_0^2}{2\mu g} = \frac{4\times 4}{2\times 0.2\times 10} = 4m$$

8. (a, b, c, d)

Let α be the angle of incidence at the quarter cylinder.



$$\tan\alpha = \frac{h}{\sqrt{R^2 - h^2}}$$

The greater the value of h, the largest is angle of incidence. A is correct.

For, $r = 90^0$ the angle of incidence for the ray shown in the figure the critical angle for total internal reflection.

$$\sin C = \frac{1}{\mu} = \frac{1}{\sqrt{2}}, \alpha = C = 45^0$$

Then $\alpha = \sin 45^0 = \frac{h}{R} = \frac{(2\sqrt{2})}{\sqrt{2}} cm = 2cm$

$\therefore$ B is correct

$$\cos\alpha = \frac{R}{(R+x)} = \frac{1}{\sqrt{2}}$$

$$R\sqrt{2} = R + x$$

$$x = R(\sqrt{2-1}) = 2\sqrt{2}(\sqrt{2}-1)cm$$

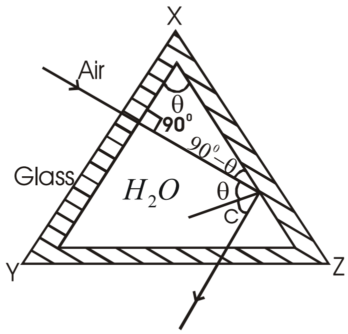
$$= (4 - 2\sqrt{2}) \simeq 1.17cm$$

$\therefore$ C is correct

To find the farthest point of the light patch consider the cylinder close to the table as a plane-convex lens with focal length

$f = \frac{R}{(\mu - 1)} = \frac{2\sqrt{2}}{(\sqrt{2}-1)} \neq$ infinity.

9.(b) For the surace XZ, the angle of incidence $= \theta$



Refractive index of the H_2O with respect to glass

$$_{H_2O}\mu_{glass} = \frac{\mu_{glass}}{\mu_{H_2O}} = \frac{3}{2} \times \frac{3}{4} = \frac{9}{8}$$

For water-glass interface

$$\sin C_1 = \frac{1}{_{H_2O}\mu_{glass}} = \frac{8}{9} \Rightarrow C_1 = 62.7^0$$

For glass-air interface

$$\sin C_2 = \frac{1}{_{glass}\mu_{air}} = \frac{1}{\frac{3}{2}} = \frac{2}{3} \Rightarrow C_2 41.8^0$$

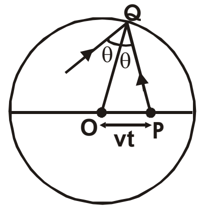
$\therefore C_2 < C_1$

Total internal reflection will occur at the glass-air interface if $\sin\theta > \frac{2}{3}$

10. Thin dark ring will be visible if ray from source gets toatl internal reflection from the spherical shell.

Let the source at any instant be at point P. Then at point Q Ray will be totally reflected if θ is equal to or greater than critical angle. If QP is equal to x, then

$$z = \cos\theta = \frac{R^2 + x^2 - v^2t^2}{2Rx}$$



For θ to be maximum

$$\frac{dz}{dx}=\frac{2x(2Rx)-2R\left(R^2+x^2-v^2t^2\right)}{2Rx}=0$$

$$\Rightarrow x=\sqrt{R^2-v^2t^2}$$

So, $\cos\theta=\frac{2\left(R^2-v^2t^2\right)}{2R\sqrt{R^2-v^2t^2}}=\frac{\sqrt{R^2-v^2t^2}}{R}$

For no light to come out, $\sin\theta\geq\frac{1}{\sqrt{2}}$

or $\theta\geq 45^0$

For minium time, $\cos\theta=\frac{1}{\sqrt{2}}$

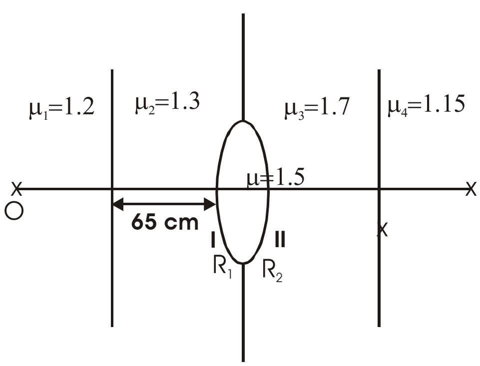
or $\frac{1}{\sqrt{2}}=\frac{\sqrt{R^2-v^2t^2}}{R}$

or $vt=\frac{R}{\sqrt{2}}\Rightarrow t=1s$

11. Distance of the object from the lens

$$u=-\left[65+60\times\frac{1.3}{1.2}\right]=-130\,cm$$

For the refraction at the first surface



$$\frac{u}{v_1}-\frac{\mu}{u}=\frac{\mu-\mu_2}{R_1}$$

$$\frac{1.5}{v_1}-\frac{1.3}{-130}=\frac{1.5-1.3}{10}$$

$$\Rightarrow\frac{1.5}{v_1}=\frac{0.2}{10}-\frac{1.3}{130}$$

$$=\frac{2}{100}-\frac{1}{100}=\frac{1}{100}$$

$$\Rightarrow v_1=150\,cm$$

For the refraction at the second surface

$$\frac{\mu_3}{v_2}-\frac{\mu}{v_1}=\frac{\mu_3-\mu}{R_2}$$

$\mu=1.5,\quad \mu_3=1.7,\quad v_1=+150cm,$

$R_2=-60cm$

$$\frac{1.7}{v_2}-\frac{1.5}{150}=\frac{1.7-1.5}{-60}$$

$$\frac{1.7}{v_2}=\frac{1}{100}-\frac{1}{300}=\frac{2}{300}$$

$$\Rightarrow v_2=255cm$$

Distance of the image from the lens

$$=85+(255-85)\times\frac{1.15}{1.7}$$

$$=200\,cm=2m$$

12. If the particles collide after t seconds, then

$$9t+\left(v_0\cos 37^0\right)t=x \qquad ....(1)$$

$$=50m$$

$$9t+\left(v_0\sin 37^0 t-\frac{1}{2}gt^2\right)+\frac{1}{2}gt^2=y \qquad ....(2)$$

$$=24m$$

Solving equations (1) and (2),

$t=2s\Rightarrow v_0=20m/s$

The horizontal distance of the point of collision from $A=\left(v_0\cos 37^0\right)t$

$= 20 \times \frac{4}{5} \times 2$

$= 32m$

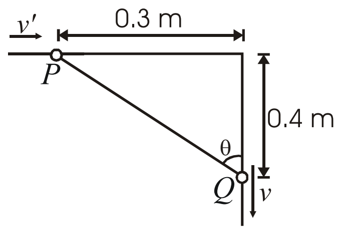
Vertical height of the point of collision from

$A = \left(v_0 \sin 37^0\right)t - \frac{1}{2}gt^2$

$= 20 \times \frac{3}{5} \times 2 - \frac{1}{2} \times 10 \times 4 \ = 4m$

13. (3)

Relative velocity of P and Q along PQ should be zero.



Hence, $v' \sin\theta = v\cos\theta$

$v' = v\cot\theta$

$v' = \frac{4}{3}v$ (i)

Applying conservation of mechanical energy we have : decrease in potential energy of Q = increase in kinetic energy of both

$\therefore (1)(10)(0.1) = \frac{1}{2} \times 1 \times v^2 + \frac{1}{2} \times 1 \times \left(\frac{4}{3}\right)^2 (v)^2$

or $18 = 9v^2 + 16v^2$

$\therefore \ v = \frac{3\sqrt{2}m}{5}\frac{m}{s}$ $\therefore \ v' = \frac{4\sqrt{2}}{5}\frac{m}{s}$

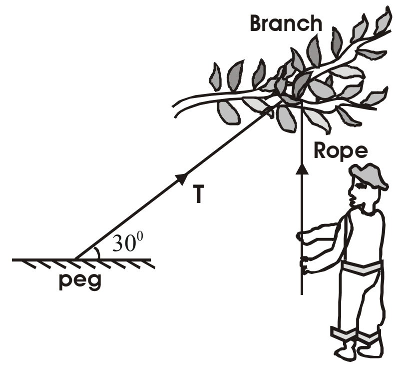
14. $\int_0^x \left(a - bx - \mu gm\right) \cdot dx = 0$

$\therefore \ ax - \frac{bx^2}{2} - \mu mgx = 0$

or $x = \frac{2\left(a - \mu mg\right)}{b}$

Putting the values we get $x = 5m$.

15. Let T be the tension in rope. The upward force on the peg



$= T\sin 30^0 = \frac{T}{2}$.

The maximum tension that will not detached the clamp from the ground is hence given by

$\frac{T}{2} = 360N$

$T = 720N$

The acceleration of the man in the upward direction is a.

The equation of motion of the man is

$T - 600 = 60a$

$\therefore \quad a = \frac{720 - 600}{60} = 2m/\sec^2$

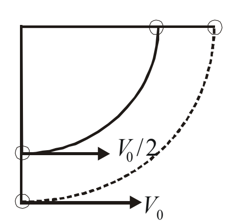
16. (12)

Considering conservation of energy between vertical position and horizontal position.

In this case, the two masses will have the same angular velocity but different linear velocities.

$\therefore \frac{V_0}{l} = \frac{V'}{l/2}$

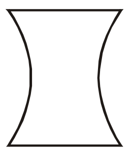
or $V' = \frac{V_0}{2}$ where V' is the linear velocity of the mass at middle.



$$mgl + mg\frac{l}{2} = \frac{1}{2}mV_0^2 + \frac{1}{2}m\left(\frac{V_0}{2}\right)^2$$

$$\frac{3}{2}mgl = \frac{5}{8}mV_0^2 \quad \text{or} \quad V_0 = \sqrt{\frac{12}{5}gl}$$

17. (4) Let f be the focal length of lens



$$\frac{1}{f} = \left(\frac{\mu_{Lens}}{\mu_{medium}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$\mu_{Lens} = 1.2, \quad \mu_{medium} = 1.6$,

$R_1 = -20\,cm, \quad R_2 = 20\,cm$

$$\frac{1}{f} = \left(\frac{1.2}{1.6} - 1\right)\left[\frac{1}{-20} - \frac{1}{20}\right]$$

$$= \left(-\frac{0.4}{1.6}\right)\left(-\frac{2}{20}\right) = \frac{1}{40}$$

Minimum distance required $= 40cm$

18. (A)$\rightarrow$p; (B)$\rightarrow$r; (C)$\rightarrow$r; (D)$\rightarrow$p

For given graph :

$a =$ Slope of $v - t$ graph

= negative but constant.

19. (A) $\rightarrow$ p,s; (B) $\rightarrow$ s; (C) $\rightarrow$q,s; (D)$\rightarrow$q,s

CHEMISTRY

20.(a) anode : $\frac{1}{2}H_2 \longrightarrow H^+ + 1e^-$

cathode : $AgCl + 1e^- \longrightarrow Ag + Cl^-$

net : $\frac{1}{2}H_2 + AgCl \xrightarrow{1e^-} H^+ + Ag + Cl^-$

$$E_{cell} = +0.222 + \frac{0.059}{1}\log\frac{1}{[H^+][Cl^-]}$$

$$= +0.222 + 0.059\log\frac{[OH^-]}{[10^{-14}][Cl^-]}$$

= + 0.222 + 0.059 (14)

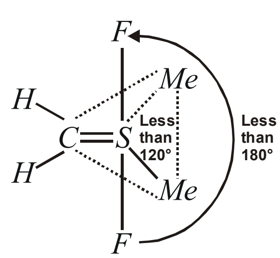
= + 1.048 volt

21.(a) I - Non polar structure.

II - Unlike charges closes to each other.

III - Unlike charges away from each other.

22.(b)



Hybridisation : sp^3d

Both $C-H$ bonds are in axial plane

Nodal plane of π bond lies in axial plane

π bond is formed by $p\pi - d\pi$ overlapping.

23. (d) (a) $H^+ + H_2O \rightleftharpoons H_3O^+$

(b) $H^+ + 4H_2O \rightleftharpoons H_9O_4^+$

(c) $H^+ + 2H_2O \rightleftharpoons H_5O_2^+$

24. (a, c)

25. (b,d)

$H_2(g) \rightarrow 2H^+(aq) + 2e^-$

Standard hydrogen electrode

$\frac{1}{2}Hg_2Cl_2(s) + e^- \rightarrow Hg(l) + Cl^{\ominus}(aq)$

Calomel electrode

26.(c) Due to much lower freezing point of eutectic muxture of $CaCl_2 / H_2O$.

27.(b,c,d)

109% suggests total mass of pure H_2SO_4 present in 100g oleum.

When 9g of H_2O is added to 100g oleum, it amounts total 109g.

H_2O combines with free SO_3 present in oleum.

$$H_2O + SO_3 \longrightarrow H_2SO_4$$

$$mol = 18g \quad 1mol = 80g$$

9g combines with 40g

Hence, 100g of oleum contains 40g of SO_3

Hence, 40% free SO_3.

28. (a,d)

Minimum boiling point azeotopic mixture is given by solution whose boiling point is less than either of the two pure components or composition of a non-ideal solution showing positive deviation for which the vapour pressure is maximum.

As $CCl_4 +$ chloroform and ethyl alcohol + water show positive deviation form minimum boiling point azeotropic mixture.

29. (100)

For $NaCl$ type structure

$$\frac{r_{C^+}}{r_{A^-}} = 0.414 - 0.737$$

For smallest cation

$$\frac{r_{C^+}}{r_{A^-}} = 0.414$$

$$\frac{r_{C^+}}{241.5} = 0.414$$

$$r_{C^+} = 0.414 \times 241.5 = 99.90 \approx 100.00$$

$$r_{C^+} = 100$$

30. (2)

KCl in 100g water 10.0 g

in 1000 g water = 100 g

m (molality) = $\frac{100}{74.55} = 1.34$

$$i = \frac{4.5^0 C}{(1.86^0 C/m)(1.34m)}$$

$$i = 1.8$$

31. (27min. or 1635)

$$C_4H_8 \xrightarrow{\Delta} 2C_2H_4$$

at $t = 0$ $\quad a \quad 0$

at $t = t$ $\quad (a - x) \quad 2x$

When $\frac{2x}{a-x} = 1$

it means $x = \frac{9}{3}$

$$t = \frac{2.303}{k} \log \frac{a}{a-x}$$

or $$t = \frac{2.303}{2.48 \times 10^{-4}} \log \frac{a}{a - \frac{a}{3}}$$

$t = 1635.2s = 27.25$ min.

32. (6)

33. (4 electron volt)

Stopping potential is the maximum kinetic energy of photoelectron :

$$KE(\text{maximum}) = eV = 0.5 \times 1.6 \times 10^{-19}$$

$$= 8 \times 10^{-20} J$$

Energy of photon

$$= \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{253.7 \times 10^{-9}}$$

$$= 7.835 \times 10^{-19} J$$

Work function

$$= 7.835 \times 10^{-19} - 8 \times 10^{-20}$$

$$= 7.035 \times 10^{-19}$$

$$= 4.4 eV$$

34. (46)

E.N. on pauling scale $= \frac{7}{2.8} = 2.5$

E.N. on pauling scale for 2nd element

$= \frac{1.4}{2.8} = 0.5$

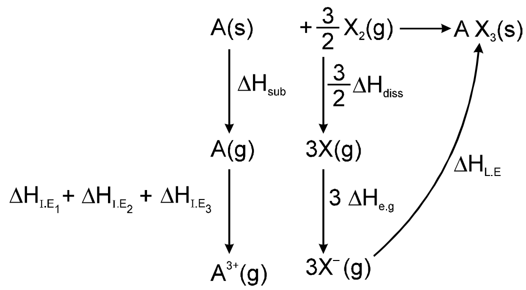
Electronegativity difference $\Delta = 2.5 - 0.5 = 2$

$\therefore$ % ionic character = $16\Delta + 3.5\,\Delta^2$

$= 16 \times 2 + 3.5 \times 2^2 = 32 + 14 = 46\%$

Ans. 46

35. (90)



$$\Delta H_f = \Delta H_{sub} + \Delta H_{I.E_1} + \Delta H_{I.E_2} + \Delta H_{I.E_3}$$

$$+ \frac{3}{2}\Delta H_{diss} + 3\,\Delta H_{e.g} + \Delta H_{L.E}$$

$$= 100 + 60 + 150 + 280 + \frac{3}{2} \times 80 - 3 \times 110 - 470$$

$$= 100 + 60 + 150 + 280 + 120 - 330 - 470$$

$$= -90 \text{ Kcal/mol}$$

Ans. 90

36. (5)

Given,

c l + e $\longrightarrow$ Cl^- + 3.7 eV

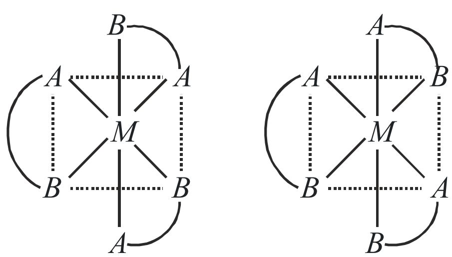
35.5 3.7×23.06 kcal

Energy released for conversion of 2 g gaseous chlorine into Cl^- ions

$$= \frac{3.7 \times 23.06}{35.5} \times 2 = 4.8 \text{ kcal}$$

37. (A) $\rightarrow$ p,q ; (B) $\rightarrow$ p,r ; (C) $\rightarrow$ r,s ; (D) $\rightarrow$ p,q,r,s

(p)



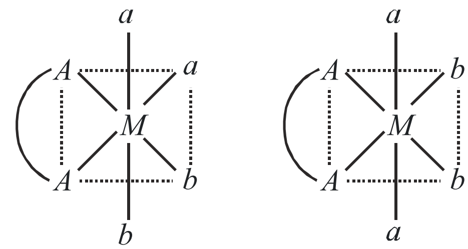
Cis : optically active   Trans : optically active

Number of Geometrical isomer = 2

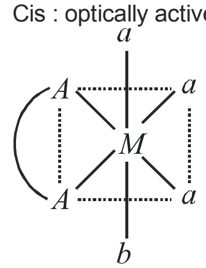
Number of optical isomer = 4

Total number of space / stereo isomers = 4

(q)



Cis : optically active   Trans : optical isomer



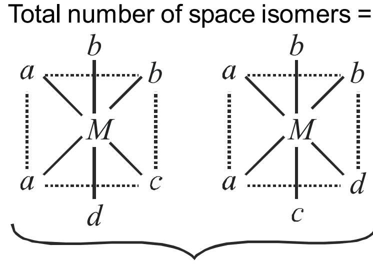
Trans : optical isomer

Number of Geometrical isomer = 3

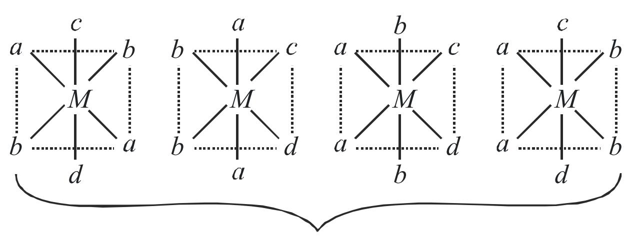
Number of optical isomer = 2

Total number of space isomers = 4

(r)



Cis : optically active



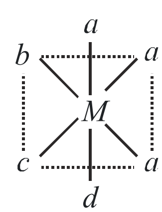
Trans : optically inactive

Number of Geometrical isomer = 6

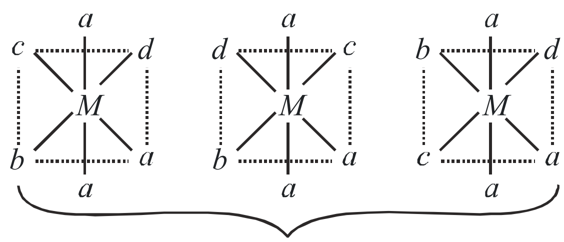
Number of optical isomer = 4

Total number of space/stereo isomers = 8

(s)



Cis : optically active

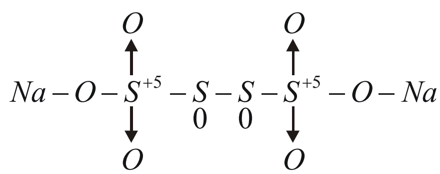


Trans : optically inactive

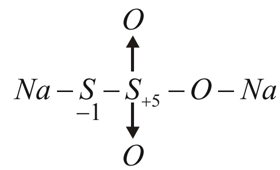
Number of Geometrical isomer = 4
Number of optical isomer = 2
Total number of space/stereo isomers = 5

38. (A) $\rightarrow$ q,r ; (B) $\rightarrow$ p,q ; (C) $\rightarrow$ p,s ; (D) $\rightarrow$ r

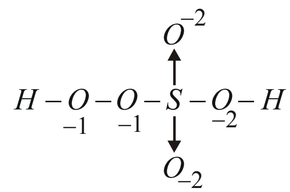
$Na_2S_4O_6$:



$Na_2S_2O_3$:



H_2SO_5 :



$Fe(CO)_5 : CO$ is neutral, oxidation number =0

$Fe = 0$

MATHEMATICS

39.(a) $f(x) = \sqrt{2}\sin\left(x + \frac{\pi}{4}\right) + 2\sqrt{2}$ or

$f(x) = \sqrt{2}\cos\left(x - \frac{\pi}{4}\right) + 2\sqrt{2}$

$\Rightarrow Y = \left[\sqrt{2}, 3\sqrt{2}\right]$

and $X = \left[-\frac{3\pi}{4}, \frac{\pi}{4}\right]$ or $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$

40.(c) Let $Z = 2e^{i\theta}$, $0 \le \theta \le \pi/2$

and $\omega = x + iy$

$\omega = \dfrac{2e^{i\theta}+1}{2e^{i\theta}+2} = 1 - \dfrac{1}{2(e^{i\theta}+1)}$

$= 1 - \dfrac{1}{2e^{i\theta/2}(e^{i\theta/2}+e^{-i\theta/2})}$

$= 1 - \dfrac{e^{-i\theta/2}}{2(e^{i\theta/2}+e^{-i\theta/2})}$

$= 1 - \dfrac{e^{-i\theta/2}}{2.2\cos(\theta/2)}$

$= 1 - \dfrac{\cos(\theta/2) - i\sin(\theta/2)}{4\cos(\theta/2)}$

$= 1 - \dfrac{1}{4}\left(1 - i\tan\dfrac{\theta}{2}\right)$

$x + iy = \dfrac{3}{4} + \dfrac{i}{4}\tan\dfrac{\theta}{2}$

$\Rightarrow$ Re(ω) lies on the line $x = \dfrac{3}{4}$ with $y \in [0, 1/4]$

41.(bc) Since matrix A is skew-symmetric,

$\therefore$ $|A| = 0$

$\therefore$ $|A^4 . B^3| = 0$

42.(d) $\left(\hat{i} + \hat{j}\right) \times \left(\hat{j} + \hat{k}\right) = \hat{i} - \hat{j} + \hat{k}$

$\Rightarrow$ unit vector perpendicular as to the plane of $\hat{i} + \hat{j}$ and $\hat{j} + \hat{k}$ is $\dfrac{1}{\sqrt{3}}\left(\hat{i} - \hat{j} + \hat{k}\right)$

similarly other two unit vectors are

$\frac{1}{\sqrt{3}}\left(\hat{i}+\hat{j}-\hat{k}\right)$ and $\frac{1}{\sqrt{3}}\left(-\hat{i}+\hat{j}+\hat{k}\right)$

$\Rightarrow$ v = $\left[\hat{n}_1\ \hat{n}_2\ \hat{n}_3\right]$ = $\frac{1}{3\sqrt{3}}\begin{vmatrix}1 & -1 & 1\\ 1 & 1 & -1\\ -1 & 1 & 1\end{vmatrix}$ = $\frac{4}{3\sqrt{3}}$

Alternatively :

Let $\vec{a}=\hat{i}+\hat{j}$; $\vec{b}=\hat{j}+\hat{k}$ & $\vec{c}=\hat{k}+\hat{i}$.

Now $\left[\vec{a}\times\vec{b}\ ,\ \vec{b}\times\vec{c}\ ,\ \vec{c}\times\vec{a}\right]$

= $\left[\vec{a}\ \vec{b}\ \vec{c}\right]^2$ = $\begin{vmatrix}1 & 1 & 0\\ 0 & 1 & 1\\ 1 & 0 & 1\end{vmatrix}^2$

= $\left[1(1)-1(0-1)\right]^2$ = 4

Hence actual volume with unity vectors

= $\frac{4}{\left|\vec{a}\times\vec{b}\right|\left|\vec{b}\times\vec{c}\right|\left|\vec{c}\times\vec{a}\right|}$

Now $\left|a\times\vec{b}\right|$ = $\sqrt{4-1}$ = $\sqrt{3}$ etc

V_{actual} = $\frac{4}{3\sqrt{3}}$

43. (c, d)

y = $\frac{x+3}{x+1}$ > 0

$\Rightarrow$ x < –3 or x > –1

as x $\to$ –3, y $\to$ 0 ; x $\to$ ∞, y $\to$ 1

range (0, 1) $\cup$ (1, ∞)

44. (b, d)

clearly at x = $\frac{3\pi}{2}$, f(x) is continuous but not differentiable

at x = $\frac{\pi}{4}$, f ′ = 0, at x = $\frac{\pi}{2}$, f’ = –1

and f’ $\left(\frac{\pi}{3}\right)$ does not exist

45. (a,d)

f(x) = $\begin{cases} ax^2+bx & \text{for } -1<x<1\\ \frac{a-b-1}{2} & x=-1\\ \frac{a+b+1}{2} & x=1\\ \frac{1}{x} & \text{for } x>1 \text{ or } x<-1\end{cases}$

for continuity at x = 1

a + b = 1 .... (A)

for continuity at x = – 1

$\Rightarrow$ a – b = – 1 .... (B)

hence a = 0 and b = 1

46. (a,b)

|w| = $\frac{|v||u-z|}{|\bar{u}z-1|}$ = $\frac{|u-z|}{|\bar{u}z-1|}$

let | w | $\le$ 1

$\Rightarrow$ | u – z | $\le$ $|\bar{u}z-1|$

$\Rightarrow$ $|u-z|^2 \le |\bar{u}z-1|^2$

$(u-z)(\bar{u}-\bar{z}) \le (u\bar{z}-1)(\bar{u}z-1)$

This simplifiese to

$\underbrace{(|u|^2-1)}_{(-ve)}$ $(|z|^2-1) \ge 0 \Rightarrow |z|^2-1 \le 0$

$\Rightarrow$ (A) and (B) are answers]

47. (c)

For the equation to have four real roots ,the line y = k must intersect $|ax^2+bx+c|$ at four points $\Rightarrow$ D > 0 and $k\in\left(0, \frac{-D}{4a}\right)$ For the equation to have three real roots ,the line y = k must intersect $|ax^2+bx+c|$ at three points $\Rightarrow$ D > 0 and $k=\frac{-D}{4a}$

For the equation to have two real roots ,the line y = k must intersect $|ax^2+bx+c|$ at two points

$\Rightarrow$ $k>\frac{-D}{4a}$

48. 0

49. replace $x \to 1/x$ and solve to get

$$f(x) = \frac{x+1}{2}$$

$$f(2009) = \frac{2009+1}{2} = 1005$$

50. $$f(x) = \begin{cases} ax^3 + b, & 0 \le x \le 1 \\ 2\cos \pi x + \tan^{-1} x, & 1 < x \le 2 \end{cases}$$

$$f'(x) = \begin{cases} 3ax^2 & 0 < x < 1 \\ -2\pi \sin \pi x + \frac{1}{1+x^2}, & 1 < x < 2 \end{cases}$$

As the function is differentiable in [0, 2]

$\Rightarrow$ function is differentiable at x = 1

$\therefore f^1(1^-) = f^1(1^+)$

$\Rightarrow 3a = \frac{1}{2} \Rightarrow a = \frac{1}{6}$

Function will also be continuous at x = 1

$\therefore \lim_{x\to 1^{1-}} f(x) = \lim_{x \to 1^{1+}} f(x)$

$\Rightarrow a + b = -2 + \frac{\pi}{4}$

$\therefore b = -2 - \frac{1}{6} + \frac{\pi}{4} - \frac{13}{6}$

$\Rightarrow k_1 = 6$ and $k_2 = 12 \Rightarrow k_1^2 + k_2^2 = 180$

51.

$$f(n) = \frac{1}{\sqrt{n^2}} + \frac{1}{\sqrt{(n^2+2)}} + \frac{1}{\sqrt{(n^2+2)}} + ... + \frac{1}{\sqrt{n^2+2n+1}}$$

terms of the sequence are decreasing and number of terms are (2n + 2)

$$\frac{2n+2}{\sqrt{n^2+2n+1}} \le f(n) \le \frac{2n+2}{\sqrt{n^2}}$$

Now $$\lim_{n\to\infty} \frac{2(n+1)}{\sqrt{n^2+2n+1}} = \lim_{n\to\infty} \frac{2n\left(1+\frac{1}{n}\right)}{n\sqrt{1+\frac{2}{n}+\frac{1}{n^2}}}$$

$$= \lim_{n\to\infty} \frac{2(n+1)}{\sqrt{n^2}} = \lim_{n\to\infty} \frac{2(n+1)}{n} = 2$$

$$\therefore \lim_{n\to\infty} f(x) = 2$$

52. $|A| = -1$

$\Rightarrow$ $|\text{adj } A| \therefore |A|^{3-1} = |A|^2 = 1$

$\Rightarrow$ $|\text{adj (adj } A)| = |\text{adj } A|^2 = 1$

....

....

....

$\Rightarrow$ $|\text{adj (adj} \ldots \text{adj } A)| = 1$

53. $\vec{r} \times \vec{b} = \vec{a} \Rightarrow \vec{r}$ is perpendicular to $\vec{a}$ and hence lies in plane of $\vec{b}$ and $\vec{a} \times \vec{b}$

$\Rightarrow$ $\vec{r} = x\vec{b} + y(\vec{a} \times \vec{b})$

taking cross product with $\vec{b}$

$\vec{a} = y(\vec{a} \times \vec{b}) \times \vec{b}$

$$y = \frac{-1}{|\vec{b}|^2}$$

$$9 = \frac{1}{|\vec{b}|^2} \left(|\vec{a}||\vec{b}|\right) \Rightarrow \frac{|\vec{a}|}{|\vec{b}|} = 9$$

54. $z_1\bar{z}_1 = 1; \quad z_2\bar{z}_2 = 1; \quad z_3\bar{z}_3 = 1$

given $z_1 + z_2 + z_3 = 0$

$\Rightarrow$ $\bar{z}_1 + \bar{z}_2 + \bar{z}_3 = 0$

hence $(z_1 + z_2 + z_3)^2 = 0$

$$\sum z_1^2 + 2z_1z_2z_3 \left[\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right] = 0$$

hence $\sum z_1^2 + 2z_1z_2z_3 [\bar{z}_1 + \bar{z}_2 + \bar{z}_3] = 0$

$\therefore$ $\sum z_1^2 = 0$

55. 1

56. (A) $\to$ (s); (B) $\to$ (p); (C) $\to$ (p); (D) $\to$ (r)

(A) $$\lim_{x\to 1} \frac{x^3 + 2x^2 + x + 21}{x^2 + 2x + 1} = \frac{25}{4}$$

$$\lim_{x\to 1} \frac{1 - \cos(x-1)}{(x-1)^2} = \lim_{y\to 0} \frac{1-\cos y}{y^2}$$

$$\lim_{y\to 0} \frac{2\sin^2 \frac{y}{2}}{y^2} = \lim_{y\to 0} \frac{1}{2}\left(\frac{\sin \frac{y}{2}}{\frac{y}{2}}\right)^2 = \frac{1}{2}$$

$\therefore$ required limit $= \left(\frac{25}{4}\right)^{1/2} = \frac{5}{2}$

(B) $\lim\limits_{x\to\frac{\pi}{2}} \frac{\sin^2 x}{\cos^2 x}\left\{\frac{\left(2\sin^2 x+3\sin x+4\right)-\left(\sin^2 x+6\sin x+2\right)}{\sqrt{2\sin^2+3\sin x+4}+\sqrt{\sin x^2+6\sin x+2}}\right\}$

$= \frac{1}{\sqrt{9}+\sqrt{9}} \lim\limits_{x\to\frac{\pi}{2}} \frac{\sin^2 x}{1-\sin^2 x}\left(\sin x-2\right)\left(\sin x-1\right)$

$= \frac{1}{6}\cdot(1)\lim\limits_{x\to\frac{\pi}{2}} \frac{\sin x-2}{-\left(\sin x+1\right)} \quad = \frac{1}{6}\frac{(1)(-1)}{-2} = \frac{1}{12}$

(C) $\frac{a-a}{g(a)f(a)-f(a)g(a)} = \frac{0}{0}$ form

By L' Hospital rule,

$\lim\limits_{x\to a} \frac{1}{g'(x)f(a)-g(a)f'(x)} = \frac{1}{g'(a)f(a)-g(a)f'(a)}$

$= \frac{1}{4\times 3-(0)2} = \frac{1}{12}$

(D) $\lim\limits_{x\to 0} \frac{5\cos x+\frac{5}{2}x\sin x-5}{x^4} \qquad \left(\frac{0}{0}\right)$ form

$= \lim\limits_{x\to 0} \frac{-5\sin x+\frac{5}{2}x\sin x+\frac{5}{2}x\cos x}{4x^3}$

$= \lim\limits_{x\to 0} \frac{\frac{5}{2}\left(x\cos x-\sin x\right)}{4x^3}$

$= \lim\limits_{x\to 0} \frac{5}{2} - \frac{\left(x\sin x\right)}{12x^2} \qquad = \frac{-5}{24}$

57. (A) $\to$ (s); (B) $\to$ (r); (C) $\to$ (q); (D) $\to$ (p)

(A) $\alpha^2+2\alpha+3=0$

$\alpha^3-2\alpha+3\alpha=0 \Rightarrow \alpha^3=2\alpha-3\alpha$

One root $= 2\alpha^2-3\alpha-3\alpha^2+5\alpha-2$

$= -\left(\alpha^2-2\alpha\right)-2$

$= -\left(-3\right)-2=1$

Other root $= 2\beta^2-3\beta-\beta^2+\beta+5$

$= \beta^2-2\beta+5$

$= -3+5=0$

Required equation is $x^2-3x+2=0$

(B) Roots are $\frac{1}{2},\frac{3}{4}$

$\therefore \alpha=\frac{1}{2}; \beta^2=\frac{3}{4}\left(\because \beta^2>\frac{1}{2}\right)$

$\Rightarrow \beta=\frac{\sqrt{3}}{2}$

$\alpha+i\beta=\frac{1}{2}+i\frac{\sqrt{3}}{2}=e^{i\frac{\pi}{3}}$

$\left(\alpha+i\beta\right)^{100}=e^{i\frac{\pi}{3}\cdot 100}=e^{33i\pi}\cdot e^{i\frac{\pi}{3}\cdot}$

$=-e^{i\frac{\pi}{3}}$

Similarly, $\left(\alpha+i\beta\right)^{100}=-e^{-i\frac{\pi}{3}\cdot}$

Sum $= -\left(2\cos\frac{\pi}{3}\right)=-2\left(\frac{1}{2}\right)=-1$

Product $=1$

Required equation is $x^2+x+1=0$

(C) $\alpha+\beta=-6; \alpha\beta=5$

$\left(\alpha+\beta\right)=36$

$\left(\alpha-\beta\right)^2=\left(\alpha+\beta\right)^2-4\alpha\beta$

$=36-20=16$

$\left(\alpha-\beta\right)^2=16$

Required equation is $x^2-52x+576=0$

(D) $-47+8\sqrt{-3}=\sqrt{a+ib}$

$a^2-b^2=-47;\ 2iab=8\sqrt{3i}$

$\Rightarrow a=1;\ b=4\sqrt{3}$

One root $=1+i4\sqrt{3}$

Other root $=1-i4\sqrt{3}$

Sum $=2$

Product $=1-16\times 3\left(-1\right)=49$

Required equation is $x^2-2x+49=0$